Journal of Computational Physics 347 (2017) 78–98

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

A locally conservative stabilized continuous Galerkin finite element method for two-phase flow in poroelastic subsurfaces

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ARTICLE INFO

Article history:
Received 12 October 2015
Received in revised form 4 May 2017
Accepted 8 June 2017
Available online 23 June 2017

Keywords:
Poroelastic
Geomechanic
Multiphase flow
Local conservation
Post-processing
Stabilized CGFEM

ABSTRACT

We study the application of a stabilized continuous Galerkin finite element method (CGFEM) in the simulation of multiphase flow in poroelastic subsurfaces. The system involves a nonlinear coupling between the fluid pressure, subsurface's deformation, and the fluid phase saturation, and as such, we represent this coupling through an iterative procedure. Spatial discretization of the poroelastic system employs the standard linear finite element in combination with a numerical diffusion term to maintain stability of the algebraic system. Furthermore, direct calculation of the normal velocities from pressure and deformation does not entail a locally conservative field. To alleviate this drawback, we propose an element based post-processing technique through which local conservation can be established. The performance of the method is validated through several examples illustrating the convergence of the method, the effectivity of the stabilization term, and the ability to achieve locally conservative normal velocities. Finally, the efficacy of the method is demonstrated through simulations of realistic multiphase flow in poroelastic subsurfaces.

1. Introduction

When a mechanical stress such as an external load is applied to a fluid-filled porous medium, a response is triggered, thereby changing the volume fraction of the pore spaces. The nature of the response, however, is a function of the stiffness of the porous material and the behavior of the fluids that are present in the pore spaces. There are mainly two mechanisms governing this fluid-material interaction: (i) the increase in pore pressure, which results in dilation of the porous material and (ii) compression of the porous material, which causes a rise in pore pressure if the fluid is unable to escape the medium. In response, any occupying fluid will move within the medium in an attempt to balance these changes in pore pressure. Poroelasticity theory, originally motivated by the theory of consolidation, studies the simultaneous deformation of a porous solid and the flow of pore fluid. The theory has seen applications in areas of geomechanics [1], reservoir engineering [2] and more recently in biomechanics [3,4]. Some specific applications include modeling of hydraulic fracturing [5], subsidence resulting from fluid withdrawal [6], and earthquake engineering [7].

The earliest development of the theory dates back to the early mid-twenties by Terzaghi [8]. He developed a model for soil consolidation in one dimension, which was later extended and generalized to three-dimension by Rendulic in 1936 [9]. In his earlier work, Terzaghi postulated that the deformation of a porous solid is mainly attributed to rearrangement of

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http://dx.doi.org/10.1016/j.jcp.2017.06.024
0021-9991/

system particles; compression of solid particles and of fluid can practically be ignored. This assumption is a good representation of highly compressible soils such as soft sand and clay. However, it renders the application of the model insufficient for some physical problems; such as deformation of highly consolidated porous material (for instance sandstone). Terzaghi's model forms the basis of soil mechanics and has since been subjected to revisions and modifications. The earliest extension of the model to linear theory of poroelasticity to incorporate pore fluid compression and compression of particles was first developed by Biot in 1935 [9] and in 1941 [10]. Ever since, different reformulations of Biot's model of poroelasticity have emerged. Rice and Cleary's extension [11] is widely recognized in areas of geophysics. Other variants include works of Biot himself [12,13] and Verruijt [6].

The complexity of linear poroelasticity models is inherent in the coupling of the governing equations. Despite the linearity of the model itself, closed form solutions are still difficult to obtain, except for some special conditions, thereby limiting the applicability of the model. Numerical methods, such as finite element methods or central finite difference schemes are therefore relied upon for approximate solutions. However, naive utilization of these numerical techniques is well known to produce pressure solutions with non-physical oscillations.

It is well known that continuous Galerkin finite element approximations (CGFEM) with low order elements perform poorly when applied to nearly incompressible elasticity problems. In such situations, poor performance is attributable to effects of large Lamé parameter; resulting in the so-called Poisson locking. For instance, it is well-known that using the continuous piecewise linear finite element space simultaneously (an inf-sup unstable pair) for displacement-pressure subsystem causes instability. This results in non-physical oscillations in the discrete pressure solution. To ensure a stable discretization and oscillatory-free solutions, some recent studies emphasize choosing a suitable pair of finite element approximation spaces for the displacement-pressure subsystem. Popularly used stable pairs include the $(\mathcal{P}_k, \mathcal{P}_{k-1})$ Taylor–Hood elements and the $(\mathcal{P}_1 + \mathcal{B}_3, \mathcal{P}_1)$ MINI mixed finite element [14]. Despite their oscillatory-free pressure solution capability, one needs a very fine grid to guarantee a smooth pressure solution. Theoretical and numerical studies of such schemes have been widely studied in [15,16]. More recently, there has been efforts to produce non-oscillatory approximate pressure solutions with lower order finite element methods. Some newly developed techniques focus on introducing a "stabilization" term to the Galerkin formulation at the discretized level, see [17,18]. Despite the instability of the $(\mathcal{P}_1, \mathcal{P}_1)$ scheme, this artificial stabilization term has been established to add stability to the scheme [19,20]; thereby reviving its applicability to displacement-pressure problems.

The model of poroelasticity has been extensively investigated by Murad et al., see [21–23]. Although most of their recent work focused on statistical properties of poroelastic media, they have also done a lot of analytical work to validate their numerical findings. Particular to what follows in this paper is their work on two-phase flow in heterogeneous poroelastic media [21], where a family of locally conservative methods has been developed based on mixed finite elements for hydrodynamic and poromechanic equations. In their work, the Raviart–Thomas mixed finite element was applied to the hydrodynamic equation while the B-bar method is used to solve the poromechanic equation. Another distinguishing feature of their method is the decoupling of the hydrodynamic and geomechanical problems from the transport subsystem. This allows for an iterative solution of the hydrodynamic and geomechanical problems to be found at each new time level. Once the displacement is obtained at this new time level, porosity is updated based on the mass balance of the solid phase. With updated information, the transport subsystem can then be solved for updated fluid saturations.

The present investigation centers on the development of an efficient and accurate numerical method for two-phase flow in the heterogeneous poroelastic subsurface. In this regard, two main contributions are offered. Firstly, the geomechanic quantities of interest as depicted by the quasi-static Biot's consolidation model is approximated by the CGFEM using a stabilized $(\mathcal{P}_1, \mathcal{P}_1)$ element. We utilize a stabilization that involves adding a perturbation to the finite element discretization whose robustness was recently established [20]. The adoption of a stabilized $(\mathcal{P}_1, \mathcal{P}_1)$ element into settings of this kind certainly merits serious attention. This is not only due to its simplicity, but also because of the efficiency it can offer in relative comparison to other approaches. The current work serves as a jump-start for further investigations. Furthermore, in the framework of two-phase flow, it is imperative to gather derivable quantities from the geomechanics in the form of fluid and solid velocity fields that are locally conservative. The locally conservative velocity fields are input to the transport equation in which they act as a driving mechanism to move the fluids. Due to the global nature of its formulation, it is well known that approximate solutions stemming from CGFEM do not immediately render locally conservative velocity fields. As a second contribution, we propose a simple and efficient post-processing procedure to satisfy this need.

To allow for an efficient implementation, we use an operator splitting framework. The basic strategy is to decompose the coupled system into several simpler subsystems over a relatively limited range of scales. The approximate solution of the coupled system is obtained from a combination of approximate solutions for the individual subsystems. One strong motivation for using this operator splitting is that it furnishes an efficient way to use legacy methods that are suitable for specific subsystems to tackle multiphysics problems, such as the two-phase flow in the heterogeneous poroelastic subsurface. This operator splitting allows for model extensions to be incorporated into the proposed numerical framework in a manner that is relatively straightforward. As mentioned earlier, in this application the CGFEM is used for solving the geomechanic subsystem. Using a post-processing step this approximate solution is then used to update solutions of the transport equation. Furthermore, the solution of the transport equation is suitably handled by the vertex-centered finite volume discretization and upwinding technique.

The rest of this paper is organized as follows. Discussion on the mathematical model culminating in the set of governing partial differential equations is conducted in Section 2. The methodology for finding the approximate solution of the mathe-