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A geometry preserving, conservative, mesh-to-mesh isogeometric interpolation algorithm for spatial adaptivity of the multigroup, second-order even-parity form of the neutron transport equation

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ABSTRACT

In this paper a method is presented for the application of energy-dependent spatial meshes applied to the multigroup, second-order, even-parity form of the neutron transport equation using Isogeometric Analysis (IGA). The computation of the inter-group regenerative source terms is based on conservative interpolation by Galerkin projection. The use of Non-Uniform Rational B-splines (NURBS) from the original computer-aided design (CAD) model allows for efficient implementation and calculation of the spatial projection operations while avoiding the complications of matching different geometric approximations faced by traditional finite element methods (FEM). The rate-of-convergence was verified using the method of manufactured solutions (MMS) and found to preserve the theoretical rates when interpolating between spatial meshes of different refinements. The scheme's numerical efficiency was then studied using a series of two-energy group pincell test cases where a significant saving in the number of degrees-of-freedom can be found if the energy group with a complex variation in the solution is refined more than an energy group with a simpler solution function. Finally, the method was applied to a heterogeneous, seven-group reactor pincell where the spatial meshes for each energy group were adaptively selected for refinement. It was observed that by refining selected energy groups a reduction in the total number of degrees-of-freedom for the same total L_2 error can be obtained.

1. Introduction

The neutron transport equation is a partial-integro-differential equation (PIDE) based upon a linearized version of the Boltzmann transport equation; commonly used in the field of statistical mechanics to model the average statistical behaviour of interacting gases of particles, atoms or molecules. The transport equation is used to model the statistical behaviour of neutrons interacting within a host medium such as a nuclear reactor core or a radiation shield [1]. The behaviour of the neutrons is described using the angular flux which is the dependent variable in the neutron transport equation. The angular flux is a function of a seven-dimensional phase space of independent variables consisting of three dimensions for the spatial

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variation x, y and z; two dimensions for the angular variation θ and χ; one dimension for the energy variation E and finally one variable for the temporal variation t. To accurately represent the angular flux sufficient resolution is required in each of the seven dimensions of the phase space. Combined with the large and geometrically complex nuclear reactor core designs and radiation shields the solution of this equation represents a significant challenge even for contemporary state-of-the-art numerical algorithms and high performance computing (HPC) architectures.

The variation of the angular flux with energy is usually very complex and represents a key challenge when solving the neutron transport equation. This is mainly due to the complicated variation of the interaction effects of the neutrons with the host media; which is represented by the neutron interaction cross sections. This complicated energy dependence of the cross sections leads the angular flux to have a significant dependence on energy. One common approach to discretise the energy variation of the neutron transport equation is to assume that the energy variation can be approximated by integral quantities over a number of discrete energy groups each with a prescribed upper and lower energy bound, the multigroup approximation. The energy range of neutrons typically spans between less than 1 eV to several MeV depending upon the nature of the problem being solved. Therefore, potentially a significant number of energy groups may be required to represent the energy variation of the physical phenomena described by the neutron transport equation [2].

Current deterministic reactor physics codes used in the industry for core and radiation shield design such as PANTHER [3] and ATILLA [4] utilize a common spatial mesh for each energy group in order to solve the neutron transport equation. However, the spatial variation of the angular flux will, in general, vary significantly with the energy of the neutrons; as a consequence of the energy variation of the cross sections. Therefore using the same spatial mesh for each energy group will not prove optimal. In order to optimise accuracy and computational efficiency it is more desirable to use different spatial meshes for each energy group. However, this represents another series of significant challenges for the discretisation scheme in terms of conservatively interpolating source and solution functions from one spatial mesh to another as well as preserving the exact geometry of the solution domain. The challenge of interpolating the numerical representation of a function between different spatial meshes is not unique to the application of the multigroup form of the neutron transport equation. In a wider context, function interpolation is required in a diverse range of situations; for example in the coupling of a reactor physics code to a computational fluid dynamics (CFD) code [5], visualisation and comparison of data sets computed on different spatial grids and the interpolation of a solution function onto a new grid using adaptive mesh refinement (AMR) [6].

The first application of group-dependent spatial meshes was by Wang et al. who presented an adaptive mesh refinement (AMR) scheme allowing the use of unique meshes for each energy group applied to the neutron diffusion equation [7]. Their approach used hierarchically refined finite element meshes from a common coarse grid. The use of a common coarse grid limits this approach to Cartesian geometries (or geometries consisting solely of planar interfaces) or sacrificing any improvements in the representation of the geometry as the mesh is refined. In Baker et al. a method allowing for the conservative interpolation between unstructured triangular finite element meshes was presented [2] based upon the work of Farrell and Maddison [8]. This approach required the generation of a 'super-mesh' to facilitate the transfer of information from one group to another. However, the two spatial meshes required no shared underlying structure. A reduction in the total number of finite elements required to obtain the same numerical accuracy was presented. Goffin et al. also used this methodology for goal-based AMR [9]. As in Wang et al., all of the research so far in this area has been demonstrated for Cartesian geometries only.

Isogeometric Analysis (IGA) is a generalisation of the finite element method (FEM) where the mathematical description of a computer-aided design (CAD) model, typically Non-Uniform Rational B-splines (NURBS), is used as the mathematical basis for analysis [10]. By utilising a geometry description based upon the original CAD model the mesh is exact from the coarsest level and further refinement to improve the fidelity of the solution does not alter the description of the geometry in either the parametric or real space. IGA has been extended to use locally refineable T-spline patches remedying the limitation of the tensor-product refinement of the standard NURBS representation [11]. IGA has been applied to a wide variety of physics and engineering disciplines including electromagnetics [12], computational fluid dynamics (CFD) [13,14] and solid dynamics [10].

In the field of reactor physics, IGA was first applied to the mono-energetic neutron diffusion equation by Hall et al. [15]. This work was later extended by Welch et al. who applied IGA to the multigroup form of the neutron diffusion equation for a quarter-core reactor geometry [16]. In these studies it was found that the exact representation of the problem geometry significantly improved the accuracy of the obtained solution for the same computational cost of equivalent finite element calculations. It was also found that the increased basis function continuity available to B-splines and NURBS significantly reduced the required computational effort to obtain a given level of solution accuracy. In addition to applications to the neutron diffusion equation, IGA has been applied to the first-order form of the neutron transport equation with a discontinuous Galerkin (DG) spatial discretisation and S_N angular discretisation [17].

The NURBS patches used in IGA have a series of useful properties that make them suitable for use in energy-dependent mesh methods. As the geometry is exact at the coarsest level (the CAD description) and further refinements do not change the physical geometry, a NURBS patch can be refined to suit the physics of each energy group independently with each representation conforming to the original geometry. In addition, as the parametric description does not change under refinement the projective matrices required for the function projection can be efficiently computed without the need to generate a physical 'super-mesh'.