Journal of Computational Physics 347 (2017) 207–234

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

A reduced order model based on Kalman filtering for sequential data assimilation of turbulent flows

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ARTICLE INFO

Article history:
Received 21 February 2017
Received in revised form 26 June 2017
Accepted 27 June 2017
Available online 1 July 2017

Keywords:
Kalman Filter
Turbulent flows
Numerical simulation

ABSTRACT

A Kalman filter based sequential estimator is presented in this work. The estimator is integrated in the structure of segregated solvers for the analysis of incompressible flows. This technique provides an *augmented* flow state integrating available observation in the CFD model, naturally preserving a zero-divergence condition for the velocity field. Because of the prohibitive costs associated with a complete Kalman Filter application, two model reduction strategies have been proposed and assessed. These strategies dramatically reduce the increase in computational costs of the model, which can be quantified in an augmentation of 10%–15% with respect to the classical numerical simulation. In addition, an extended analysis of the behavior of the numerical model covariance Q has been performed. Optimized values are strongly linked to the truncation error of the discretization procedure. The estimator has been applied to the analysis of a number of test cases exhibiting increasing complexity, including turbulent flow configurations. The results show that the *augmented* flow successfully improves the prediction of the physical quantities investigated, even when the observation is provided in a limited region of the physical domain. In addition, the present work suggests that these Data Assimilation techniques, which are at an embryonic stage of development in CFD, may have the potential to be pushed even further using the *augmented* prediction as a powerful tool for the optimization of the free parameters in the numerical simulation.

1. Introduction

The accurate prediction of turbulent flow configurations is one of the ultimate open challenges in fluid mechanics studies. Most of industrial/environmental applications aim to provide accurate and robust estimation of aspects which are governed by turbulence statistical moments such as aerodynamic forces, transport of particles and heat exchange. Traditional investigative tools, such as experiments and numerical simulation, are not completely successful in producing robust descriptions of turbulent configurations, because of important fundamental drawbacks.

Measurements obtained via Experimental Fluid Mechanics (EFD), such as those sampled by surface sensors, provide a local description of flow dynamics. Because of the non-linear, strongly inertial behavior of the flow, the determination of a

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http://dx.doi.org/10.1016/j.jcp.2017.06.042
0021-9991/

complete map of flow behavior is problematic. Information can be reconstructed by the use of reduced-order models, such as the Proper Orthogonal Decomposition (POD) [1]. However, these approximated models usually provide an incomplete reconstruction of turbulent flows. One of the reasons is that strong non-linear interactions occur between the modeling error and the bias associated with the measurement, which is tied to epistemic uncertainties in the experimental sampling. This aspect, which is amplified by the multi-scale nature of turbulence, usually results in poor characteristics of robustness and precision of the reduced-order model.

Under this perspective, the use of Computational Fluid Dynamics (CFD) can provide more complete maps of flow characteristics, including regions of the physical domain were experimental sampling is problematic because of structural difficulties. However, numerical simulation is affected as well by errors/epistemic uncertainties. In this case, the parametric set-up of the simulation (physical characterization of the flow, boundary conditions...) can not exactly reproduce the subtle perturbations and in-homogeneity of the real flow, which are unknown a priori. This is particularly true for turbulence, where small perturbations present in the environment are amplified and they ultimately drive the evolution of the flow. In addition, owing to computational resources constraints, analyses of very high Reynolds flows are presently limited to reduced order simulations via RANS/LES approaches [2–4]. Turbulence/subgrid scale modeling is usually a main source of error in CFD, in particular because of its non-linear interaction observed with the numerical/boundary condition error. Thus, while both EFD and CFD are affected by bias, the confidence level of the results is affected by uncertainties of a completely different nature. This is the reason why the comparison of experiments and numerical results is a complex task even for the classical case of grid turbulence decay [5].

In the last decade, new methodological approaches coming from Estimation Theory (ET) have been employed by the fluid mechanics community to obtain an optimized prediction of flow configurations. ET is a branch of statistics dealing with the estimation of optimal parametric description, using data which are affected by a level of uncertainty/stochasticity [6]. In particular, Data Assimilation (DA) includes a wide spectrum of tools which aim to the estimation of an optimal state integrating a model and observations which are affected by uncertainties. They are usually referred to as *estimators*. Some studies in EFD have been proposed, where the DA tool combines experimental sampling with reduced order numerical solvers in order to provide the zero-divergence condition of incompressible flows [7,8]. Early CFD applications mainly deal with variational approaches based on the adjoint method, which has been a classical choice for meteorological studies since the 1970s [9]. These approaches are defined as an optimization problem where a given measure is minimized under the constraint of the governing equation. The resolution of this problem determines the distribution of a basis of parameters (typical choices are boundary/initial conditions) which optimizes the flow configuration. Recent applications deal both with fundamental studies [10–12] and industrial oriented analyses [13,14]. While these approaches are very precise and they allow for sensitivity analyses considering a very large number of variables, their application to turbulent flow investigation over a long observation window is problematic [14,15]. A much less commonly investigated path for CFD is represented by the use of sequential methods, which are based on Bayesian inference and they occasionally require the resolution of Riccati-type equations. Examples based on techniques such as the Kalman filter [16] or the ensemble Kalman filter [17] have been reported in the literature [18–22,12].

These techniques are showing enormous potential, because the coupling between experimental/numerical data can potentially exclude the bias which can not be identified in the two methods alone. However, reliable tools for the optimal prediction of complex flow configurations are still far out of reach, because of the level of maturity of application of these techniques in fluid mechanics analyses. In fact, these methods are still largely unexplored, in particular for the analysis of turbulent flow configurations. In the present work we propose a methodological approach (estimator) for sequential Data Assimilation, which efficiently integrates information (usually experimental data) in CFD solvers. The approach is based on a reduced order Kalman filter [16], which exploits structural characteristics of the segregated solvers commonly implemented in commercial CFD software. While similar approaches have been rigorously derived for coupled solvers for incompressible flows [23], the present model proposes practical, computational inexpensive solutions for the analysis of three dimensional turbulent flow configurations.

The paper is structured as follows. In Section 2 the state of the art is presented introducing all the background elements which synergically interact in the estimator. These elements include a description of the numerical solver as well as the Kalman Filter. In Section 3 the Kalman Filter based estimator is introduced and discussed. In Section 4 an extensive analysis of the structure of the model covariance matrix Q is performed. In section 5 the estimator is applied to the analysis of laminar flows. In particular, the property of synchronization of the model with available observation is investigated. In Section 6 the analysis is extended to turbulent flow configurations and their statistical behavior. In Section 7 future developments are discussed, including parametric optimization of the numerical model. Finally, in Section 8 conclusions are drawn.

2. Numerical & methodological ingredients: state of the art

The basic elements for the development of the reduced-order Kalman filter estimator are now introduced. This includes a description of the segregated numerical CFD solver used for the resolution of the Navier–Stokes equations, as well as fundamental elements for Kalman Filter application.