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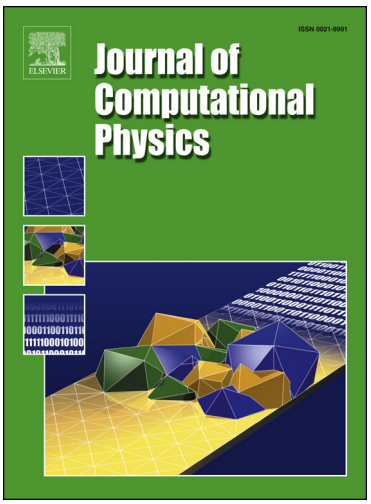

Unconditionally stable methods for gradient flow using Convex Splitting Runge–Kutta scheme

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Unconditionally stable methods for gradient flow using Convex Splitting Runge–Kutta scheme

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Abstract

We propose a Convex Splitting Runge–Kutta (CSRK) scheme which provides a simple unified framework to solve a gradient flow in an unconditionally gradient stable manner. The key feature of the scheme is a combination of a convex splitting method and a specially designed multi-stage two-additive Runge–Kutta method. Our methods are high order accurate in time and assure the gradient (energy) stability for any time step size. We provide detailed proof of the unconditional energy stability and present issues on the practical implementations. We demonstrate the accuracy and stability of the proposed methods using numerical experiments of the Cahn–Hilliard equation.

Key words: Gradient flow, Convex splitting, Gradient stability, Energy stability, Phase-field model, Cahn–Hilliard equation

1 Introduction

Gradient systems have been fundamental in the development of many important concepts in dynamical systems [1]. In particular, gradient systems are important in many phenomenological models of phase transition such as the Allen–Cahn and Cahn–Hilliard equations [2, 3]. We concentrate on a numerical method for solving the initial value problem, which can be represented as

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