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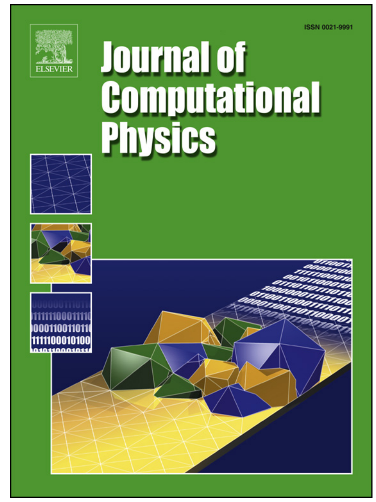

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Stochastic level-set method for shape optimisation

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We present a new method for stochastic shape optimisation of engineering structures. The method generalises an existing deterministic scheme, in which the structure is represented and evolved by a level-set method coupled with mathematical programming. The stochastic element of the algorithm is built on the methods of statistical mechanics and is designed so that the system explores a Boltzmann-Gibbs distribution of structures. In non-convex optimisation problems, the deterministic algorithm can get trapped in local optima: the stochastic generalisation enables sampling of multiple local optima, which aids the search for the globally-optimal structure. The method is demonstrated for several simple geometrical problems, and a proof-of-principle calculation is shown for a simple engineering structure.

I. INTRODUCTION

Structural optimisation aims to provide performance improvements and/or weight savings by formulating an engineering structural design problem as constrained optimisation. The class of structural optimisation that is of interest in this manuscript is shape optimisation using the level-set methodf, which systematically modifies the structural boundary, i.e. design shape, to maximise or minimise the given performance metric while satisfying one or more constraints. A related class of structural optimisation is topology optimisation in which shapes and the number of boundaries are optimised. Topology optimisation is considered the most generic form of structural optimisation since the optimal solution is the most independent of the initial solution and can offer substantial performance improvements via unintuitive and creative designs [6, 21]. To this extent, shape optimisation is an important class of structural optimisation, critically enabling topology optimisation via the level-set method. The level-set method naturally splits or merges boundaries thus topology optimisation is inherently enabled, although for the careful investigations presented in this article, we focus our attention to shape optimisation of a single external boundary of the structure.

It is well-known that many engineering applications involve optimisation problems that are *non-convex* – they support multiple locally-optimal designs, which correspond to local minima of the objective function. These local optima might be associated with different structural topologies, conflicting design requirements, and/or with numerical aspects of the (discretised) computational problem. Examples of non-convex design spaces in the engineering literature are stress constrained optimization [3] and coupled multiphysics optimization [10]. Such systems challenge conventional (deterministic) optimisation schemes, which tend to converge to *local* optima, but miss the globally-optimal structure. The purpose of this paper is to exploit an analogy between such engineering problems and statistical mechanical systems with non-convex potential energy surfaces. This motivates us to introduce a stochastic algorithm for exploring the space of possible structures, in order to sample multiple local optima and – eventually – converge to the global optimum.

In the engineering context, non-convexity and multiple optima present interesting dilemmas. If optimisation algorithms yield locally optimal structures which are much worse than the global optimum, they would not be considered useful design methods. However, generating multiple locally-optimal solutions can also provide multiple design ideas to engineers, offering a range of possible solutions with similar values of the objective function. In this case, an engineer may wish to consider several of these solutions, based on practical design requirements such as ease of manufacturing. This is particularly true when there are several designs of similar objective function and constraint values, as in [3]. Thus, methods which can sample multiple local optima are useful both for aiding the search for the global optimum, and for providing a range of potential design solutions.

A common approach to topology optimisation is to employ gradient-based nonlinear programming, which can be applied to typical engineering design problems with $10^4 - 10^6$ design variables [2, 8]. In this case, iteration of the optimiser leads to one local optimum solution. Alternative (stochastic) approaches such as evolutionary algorithms, particle swarm optimisation and simulated annealing are capable of searching for multiple potential solutions and they have been applied to topology optimisation [15, 27, 29]. However the success of such methods has been limited, partly because they do not typically take advantage of information about the gradient of the objective function. Interested readers are referred to a critical review [22] which presents an example: Topology optimisation was applied to a small problem with just 144 variables, which was solved using the non-gradient method of differential evolution. This required 15,730 function evaluations. In contrast, a gradient-based topology optimisation method – Solid Isotropic Material with Penalisation (SIMP) – converged to a slightly superior solution after just 60 function evaluations. This motivates the research question of how to explore non-convex