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MUSIC algorithm for location searching of dielectric anomalies from S-parameters using microwave imaging

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ABSTRACT

Motivated by the biomedical engineering used in early-stage breast cancer detection, we investigated the use of MUltiple SIgnal Classification (MUSIC) algorithm for location searching of small anomalies using S-parameters. We considered the application of MUSIC to functional imaging where a small number of dipole antennas are used. Our approach is based on the application of Born approximation or physical factorization. We analyzed cases in which the anomaly is respectively small and large in relation to the wavelength, and the structure of the left-singular vectors is linked to the nonzero singular values of a Multi-Static Response (MSR) matrix whose elements are the S-parameters. Using simulations, we demonstrated the strengths and weaknesses of the MUSIC algorithm in detecting both small and extended anomalies.

1. Introduction

A key application of microwave tomographic imaging is the detection of small targets, such as tumors in early-stage breast cancer, from the measured scattered field or S-parameters. Generally, breast tumors have larger permittivity than the heterogeneities in normal tissue, which in principle should allow them to be distinguished from the background medium. This is known to be a difficult problem because of its intrinsic nonlinearity and ill-posedness. However, its potential importance to human life makes it of great interest to science.

A range of techniques aimed at detecting, localizing, and characterizing small anomalies through microwave tomographic imaging have therefore been proposed. The best known technique uses a Newton-type iteration method. Many studies [1–14] have investigated iteration-based techniques, and they have been successfully used to characterize, for example, the total number, location, and outline of small anomalies associated with breast cancer. However, the success of such iteration schemes is significantly dependent on the initial prediction, which must be close to the final outcome. Iteration schemes have other limitations, including slow convergence, the local minimizer problem, the difficulty of extension to multiple anomalies, and the need for appropriate regularization. A fast technique for detecting the exact or approximate location of a target remains elusive.

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The MUltiple SIgnal Classification (MUSIC) algorithm is a well-known non-iterative imaging technique for addressing inverse scattering. This technique has been applied to a range of problems, including the identification of small targets in free space [15–17], in inhomogeneous or anisotropic media [18–20], or buried in a half-space [21–23]; detection of internal corrosion [24]; eddy-current imaging [25]; imaging of crack-like defects [26–29] and arbitrarily shaped extended targets [6,30,31]; limited-view problem detection [32,33]; and biomedical imaging [34]. A clear description of MUSIC can be found in [35]. Studies have confirmed MUSIC to be a fast, stable, and effective technique for imaging of arbitrary unknown targets in full- and limited-view inverse scattering problems. However, to the best of our knowledge, it has not yet been applied in identifying unknown dielectric anomalies from measurements of scattered or S-parameters.

In this study, we used the MUSIC algorithm operated at a single frequency to identify the location and shape of small or extended dielectric anomalies from S-parameters collected by a limited number of dipole antennas. Our approach applied the Born approximation [3] or physical factorization of the multi-static response (MSR) matrix [30]. To confirm the practicality of this approach, we established a relationship between the imaging function of MUSIC and an infinite series of Bessel functions of integer order when the total number of dipole antennas is small. The mathematical structures of MUSIC have been shown to be applicable to full- and limited-view inverse scattering problems when a large number of directions of observation are available [27,32,33]. We performed numerical simulations to confirm the relationship, using synthetic data generated by the commercial CST studio suite, which is able to produce accurate electromagnetic data. Once the target has been imaged, its shape can be used as the initial prediction and can then be made more accurate by applying the Newton-type iteration algorithm, the level-set methodology, and optimization. As the initial estimate is close to the final outcome, the computational cost is also reduced.

The rest of this paper is organized as follows. In Section 2, the forward problem is briefly discussed and the basic concept of the S-parameter is introduced. Section 3 discusses the application of the MUSIC algorithm to anomaly detection. In Section 4, we introduce the experimental design used to investigate the relationship between the imaging function of MUSIC and an infinite series of Bessel functions of integer order, when using a limited number of dipole antennas, present the analysis of the imagining function, and discuss certain phenomena noted in our results. Section 5 presents our conclusions, gives an outline of current research, and makes proposals for future studies.

2. Forward problem and S-parameter

In this section, we briefly survey the three-dimensional forward problem in the case in which an anomaly $\mathcal{A}$ with a smooth boundary $\partial\mathcal{A}$ is surrounded by N-different dipole antennas located at $\mathbf{r}'_n$, $n = 1, 2, \cdots, N$ (see Fig. 1 for description). Throughout the study, we assumed all materials and anomalies to be non-magnetic, allowing them to be classified on the basis of their dielectric permittivity and electrical conductivity at a given angular frequency ω. To reflect this, we set the magnetic permeability to be constant at every location such that $\mu(\mathbf{r}) \equiv \mu_0$. If ε_0 and σ_0 respectively denote the initial permittivity and conductivity, then by analogy, $\varepsilon_\mathcal{B}$ and $\sigma_\mathcal{B}$ are respectively the background permittivity and conductivity, and $\varepsilon_\mathcal{A}$ and $\sigma_\mathcal{A}$ are respectively those of $\mathcal{A}$. The piecewise constant permittivity $\varepsilon(\mathbf{r})$ and conductivity $\sigma(\mathbf{r})$ can then be derived as follows:

$$\varepsilon(\mathbf{r}) = \begin{cases} \varepsilon_\mathcal{A} & \text{for} \quad \mathbf{r} \in \mathcal{A}, \\ \varepsilon_\mathcal{B} & \text{for} \quad \mathbf{r} \in \mathbb{R}^3 \backslash \overline{\mathcal{A}}, \end{cases} \quad \text{and} \quad \sigma(\mathbf{r}) = \begin{cases} \sigma_\mathcal{A} & \text{for} \quad \mathbf{r} \in \mathcal{A}, \\ \sigma_\mathcal{B} & \text{for} \quad \mathbf{r} \in \mathbb{R}^3 \backslash \overline{\mathcal{A}}. \end{cases}$$

Let $\mathbf{E}_{\text{inc}}(\mathbf{r}'_n, \mathbf{r})$ be the incident electric field in a homogeneous medium due to a point current density $\mathbf{J}(\mathbf{r}'_n, \mathbf{r})$ at $\mathbf{r}'_n$ with direction $\boldsymbol{\theta}$. Analogously, let $\mathbf{E}_{\text{tot}}(\mathbf{r}, \mathbf{r}'_n)$ be the total field in the presence of $\mathcal{A}$. Then, $\mathbf{E}_{\text{tot}}(\mathbf{r}, \mathbf{r}'_n)$ satisfies

$$\frac{1}{\mu_0}\left(\nabla \times \nabla \times \mathbf{E}_{\text{tot}}(\mathbf{r}, \mathbf{r}'_n) - k^2 \mathbf{E}_{\text{tot}}(\mathbf{r}, \mathbf{r}'_n)\right) = i\omega \mathbf{J}(\mathbf{r}'_n, \mathbf{r})$$

with the transmission condition on the boundary $\partial\mathcal{A}$ and the following radiation (or open boundary) condition:

$$\lim_{|\mathbf{r}| \to \infty} \mathbf{r}\left(\nabla \times \mathbf{E}_{\text{tot}}(\mathbf{r}, \mathbf{r}'_n) - ik\hat{\mathbf{r}} \times \mathbf{E}_{\text{tot}}(\mathbf{r}, \mathbf{r}'_n)\right) = 0.$$

Here, $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$ and k denotes the wavenumber satisfying

$$k^2 = \omega^2 \mu_0 \left(\varepsilon_\mathcal{B} + i\frac{\sigma_\mathcal{B}}{\omega}\right).$$

The S-parameter (or scattering parameter) $S(a, b)$ is defined as the ratio of the output voltage (or reflected waves) at the ath antenna and the input voltage (or incident waves) at the bth antenna. In this paper, $S_{\text{scat}}(a, b)$ denotes the scattered field S-parameter obtained by subtracting the S-parameters for the total and incident fields. The presence of anomaly $\mathcal{A}$ then allows $S_{\text{scat}}(a, b)$ to be represented as follows [3]:

$$S_{\text{scat}}(a, b) \approx -\frac{k^2}{4i\omega\mu_0} \int_\mathcal{A} \left(\frac{\varepsilon(\mathbf{r})}{\varepsilon_0} - 1\right) \mathbf{E}_{\text{inc}}(\mathbf{r}'_b, \mathbf{r}) \cdot \mathbf{E}_{\text{tot}}(\mathbf{r}, \mathbf{r}'_a) d\mathbf{r}. \tag{1}$$