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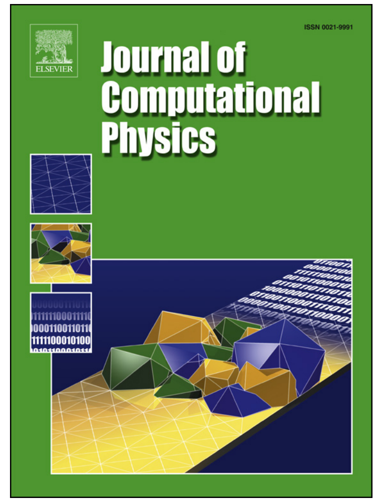

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Taylor meshless method for solving non-linear partial differential equations

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Abstract

A true meshless integration-free method based on Taylor series named Taylor Meshless Method (TMM) has been proposed recently to solve Partial Differential Equations (PDEs), where the shape functions are high degree polynomials and the discretization concerns only the boundary. With high computational efficiency and exponential convergence, the TMM has been confirmed to be very robust in linear problems, including large-scale cases. In this paper, the TMM and the Automatic Differentiation (AD) are combined with the Newton method to solve non-linear elliptic PDEs, where the AD is used to compute shape functions in a fast manner. The numerical results illustrate that the proposed algorithm is very accurate and efficient in solving non-linear elliptic problems.

Keywords: Taylor series; Multivariate algorithmic differentiation; Meshless; Newton Method.

1 Introduction

The non-linear Partial Differential Equations (PDEs) play an important role in science and engineering. To solve these non-linear PDEs, plenty of numerical methods have been developed over the past decades, such as finite difference [1, 2], finite element, finite volume [3, 4] and spectral methods [5–7]. Meshless methods based on fundamental solution [8–11], radial basis function [12–16] and moving least-squares approximations [17–21] were also applied to address non-linear PDEs. In these numerical methods, two main classes of discretization techniques were applied: Galerkin procedure and pointwise method. Galerkin-based methods may work for large-scale problems, but their requirement for integration procedures could definitely lead to high computational costs. Pointwise-based techniques avoid the drawback of integration, but they could cause ill-conditioned matrices and numerical instabilities, which make it difficult to solve large-scale problems.

This paper proposes a new numerical method to solve non-linear elliptic PDEs. The original point of the method is to compute shape functions from Taylor series as proposed in [22]. Since the PDE is solved analytically, a significant reduction in the number of degrees of freedom (DOFs) has been obtained comparing with other discretization techniques, typically a reduction ratio in the range $[20, 100]$, see [23, 24]. This strong reduction has two origins. First, because the PDE is solved analytically inside the domain, the discretization concerns only the boundary. Second, the Taylor series shows exponential convergence properties with respect to its degree and can characterize the unknown field in a large

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