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Efficient mesh motion using radial basis functions with volume grid points reduction algorithm

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A B S T R A C T

As one of the most robust mesh deformation technique available, the radial basis function (RBF) mesh deformation has been accepted widely. However, for volume mesh deformation driven by surface motion, the RBF system may become impractical for large meshes due to the large number of both surface (control) points and volume points. Surface points selection procedure based on the greedy algorithm results in an efficient implementation of the RBF-based mesh deformation procedure. The greedy algorithm could reduce the number of surface points involved in the RBF interpolation while acquire an acceptable accuracy as shown in literature. To improve the efficiency of the RBF method furthermore, an issue that how to reduce the number of the volume points needed to be moved is addressed. In this paper, we propose an algorithm for volume points reduction based on a wall distance based restricting function which is added to the formulation of the RBF based interpolation. This restricting function is firstly introduced by the current article. To support large deformation, a multi-level subspace interpolation is essentially needed, although this technique was used to improve the efficiency of the surface points selection procedure in the existed literature. The key point of this technique is setting the error of previous interpolation step as the object of current step, and restricting interpolation region gradually. Because the tolerance of the error is decreased hierarchically, the number of the surface points is increased but the number of the volume points needed to be moved is reduced gradually. Therefore, the CPU cost of updating the mesh motion could be reduced eventually since it scales with the product of these two numbers. The computational requirement of the proposed procedure is reduced evidently compared with the standard procedure as proved by some examples.

1. Introduction

Grid deformation technique is very important for the simulation of flows with a deforming boundary, such as fluid-structure interaction (FSI) problems [1–3], aerodynamic shape optimization [4] and numerical icing simulation [5]. Once the boundary is deformed, the interior points should be adjusted to avoid negative volume. Several algorithms have been developed to accomplish this task, such as the spring analogy [6,7], the linear elasticity analogy [8], Delaunay graph mapping [9], the interpolation method based on radial basis functions (RBF) [10–20], etc.

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Among these methods, grid deforming method based on the RBF has been accepted widely in the past decade due to its simplicity, robustness, flexibility and achieved mesh quality. RBF was firstly used as a technique of grid deformation by Beckert and Wendland [10]. Their work has been extended to large-scale spatial coupling problems by Ahrem and his co-workers [3]. This technique has recently used by Allen and Rendall [1,2] to develop a unified meshless approach for fluid-structure coupling and mesh motion. The RBF-based mesh deformation technique has been employed in the field of aerodynamic shape optimization by Jacobsson and Amoignon [12]. To enhance the efficiency of the RBF method, Rendall and Allen [13,14] proposed a surface points selection procedure based on a greedy algorithm. To improve the efficiency of the greedy algorithm, an incremental approach has been proposed recently by Selim and his co-workers [16]. Their method decreases the computational complexity of the greedy algorithm from $O(N_s^4)$ to $O(N_s^3)$, where N_s is the number of selected surface points. Although generally the RBF mesh deformation technique is good in preserving orthogonality, the displacements in different directions are uncoupled. Gillebaart and his co-authors [19] proposed a solution of incorporating orthogonality explicitly in the formulation by adding additional points near the boundary.

Although this approach is particularly attractive for high quality mesh deformation, it is quite expensive for large mesh size. The computational cost of the RBF method for the mesh motion is of size $N_s \times N_v$, where N_s is the number of the surface points defining the motion, and N_v is the number of the volume mesh points to be moved. Although the efficiency of the RBF technique used in the mesh deforming algorithm has been improved greatly by the greedy algorithm proposed by Allen and Rendall [13,14], the greedy algorithm itself requires very more CPU cost to select surface points. To alleviate the CPU requirement of the selection procedure itself, a multi-level subspace points selection algorithm has been developed by Wang and his collaborators [17]. It should be pointed out that the multi-level idea was employed by Wendland [21] to obtain a convergence results of RBF system before the research of Wang [17]. The works of Allen, Rendall [13,14] and Wang [17] only focused on the reduction of the number of surface points, namely N_s. To reduce time requirement of mesh motion step, a new hybrid way has been developed by Wang and his co-workers [20], which combines the advantages of Delaunay graph and RBF method. Their algorithm provides perfect performance on the save of CPU cost of mesh updating procedure.

Although the multi-level route [17] offers improved surface accuracy and reduces the CPU cost of selection procedure, it involves much more surface points. Therefore, the efficiency of volume grid deformation process is harmed by this algorithm. To solve this problem, a research [22] has been carried out by the first author of the current article. That article developed a new procedure to limit the deforming region based on the multi-level subspace points selection algorithm provided by Wang and co-workers [17], although they only use their technique to improve the efficiency of the surface points select procedure. The algorithm hierarchically restricts the volume region need to be moved at each level so that the number of volume points influenced by each level is reduced in a stepped manner. However, the previous work [22] is inconvenient and the improvement is not obvious because the restricting methodology is too complicated. This drawback motivates the present work. The principle of the algorithm of the previous and the current work [22] is that the deformation region required by the previous step is relatively large, but the number of the surface nodes chosen is small; the deformation region of next step can be set to be relatively small because the interpolation target is the error of the previous steps, although much more surface nodes are involved. The result of this algorithm is that when N_s is large, N_v is relatively small; when N_v is large, N_s is relatively small. Therefore, the computational cost of the grid motion could be saved significantly. Except the multi-level interpolation described above, another important key to construct efficient volume data reduction algorithm is how to limit the region where volume nodes need to be moved. This issue is not solved well in the previous paper [22]. Compared with the former research [22], a wall distance based restricting function has been introduced by the current article to fix up this problem by a simple way.

The remainder of this paper is organized as follows: the RBF method used in the mesh motion algorithm is summarized briefly in Section 2; the greedy algorithm and multi-level subspace technique used in the surface points selection procedure are described in Section 3; a new method to reduce the number of the volume points is developed in Section 4, which is the central contribution of the current work; numerical results are presented in Section 5 to illustrate the efficiency of the new algorithm provided by this paper; finally, conclusions are given in Section 6.

2. Radial basis function mesh motion

A radial basis function is a real-valued function whose value depends only on the distance to a support point. The general expression of the RBF is

$$\phi = \phi(\|\mathbf{r} - \mathbf{r}_i\|), \tag{1}$$

where $\mathbf{r}_i$ is coordinate of the support point, $\mathbf{r}$ is the position where the function is evaluated. Usually, the norm $\|\cdot\|$ is Euclidean distance. There are three categories of the RBF: global, local and compact. Global bases are always non-zero and grow moving away from the support point. Local bases are also always non-zero but decay moving from the origin. Compact bases decay moving from the origin like the local functions but reach zero at a finite distance. As a compact basis, the Wendland's C^2 function, which is given by

$$\phi(\eta) = \begin{cases} (1-\eta)^4(4\eta+1), & 0 \le \eta < 1 \\ 0, & \eta \ge 1 \end{cases}, \tag{2}$$