Journal of Computational Physics 348 (2017) 460–481

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Stellar surface as low-rank modification in iterative methods for binary neutron stars

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ARTICLE INFO

Article history:
Received 24 November 2016
Received in revised form 22 June 2017
Accepted 12 July 2017
Available online 18 July 2017

Keywords:
Spectral methods
Neutron stars
Numerical relativity
Equilibrium configurations
Low-rank modification
Preconditioning

ABSTRACT

We present a new multidomain spectral method for the treatment of non-spherical stellar surfaces in iterative methods for binary neutron stars. A stellar surface changes throughout the course of an iterative solution, potentially stalling the convergence. Our method affords low-complexity updates of the relevant subdomain preconditioners, thereby avoiding such stalling. Unlike current collocation (or nodal) approaches for treating surfaces (which rely on coordinate transformations to ensure that stellar surfaces arise at subdomain boundaries), our approach requires no regridding or nontrivial Jacobians. For polytropes with an equation of state specified by an integer polytropic index, our method delivers exponential accuracy with increased truncation, although for "stiff" equations of state (e.g. fractional) it suffers from the same accuracy loss as current methods. We have presented an outline of our approach before, but here present details with numerical tests.

1. Introduction

We present a strategy for the treatment of nonspherical stellar surfaces when numerically constructing binary neutron stars (BNS). Our strategy, based on modal spectral methods, is likely broadly applicable. For example, it could serve as a component in solving, via conformal methods [1–4], the constraint equations of general relativity for binary neutron star initial data (and possibly blackhole initial data with apparent horizon boundary conditions). Nonetheless, our own goal is the construction of helically symmetric BNS solutions (exact, excluding numerical error) to the matter-Einstein equations; see further remarks in Subsection 1.2. This paper focuses on the construction of equilibrium configurations in Newtonian theory, a benchmark problem [5–8] which advances us towards our longer-term goal and a stage for presenting our general strategy. While this paper only considers circular orbits and polytropic equations of state, we do not believe our approach to stellar surfaces is limited by either assumption. The central ideas of our approach have appeared before [9]. However, this paper goes beyond that account both in addressing technical details and performing numerical tests.

The key difference between our work and similar earlier approaches (described in the next subsection) is that we adopt modal spectral methods, both to achieve sparse systems of equations and because of preconditioning issues. As a second longer-term goal, we seek to exploit the sparsity associated with our modal approach, thereby achieving quantifiably fast solvers for gravitational initial data. Work towards this second goal is also incomplete. Our modal approach has been spelled out in [10–12]. While we sketch some of the key features, we do not repeat details here. Our focus is on the new development: low-rank treatment of the stellar surfaces. Nonetheless, another novelty of this work is its further development of modal spectral methods for BNS problems.

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http://dx.doi.org/10.1016/j.jcp.2017.07.026
0021-9991/

The problem of constructing (and evolving) fluid stars is not dissimilar to other problems involving fluid interfaces. Irregular interfaces and possible loss of smoothness thereon arise in myriad fluid and solid mechanical applications, for example interfacial flows involving drops and bubbles; see e.g. [13,14]. Interest often lies with the interface dynamics, as governed by the Navier–Stokes equations subject to inclusion of (singular) surface-tension forces. However, many problems in applied fluids are well-approximated by Stokes flow, for which integral formulations are well-developed; see the discussions in [15–17]. A Poisson-type problem describes Stokes flow (however interfaces are treated); likewise, the Poisson equation features prominently in this work, although its origins are different. Here it arises through inclusion of gravitational forces in the nonrelativistic Euler equations.

Numerical methods for addressing interface problems are likewise manifold, and an exhaustive review is not possible. Nonetheless, our treatment of stellar surfaces is (in concept, if not in details) reminiscent of "domain embedding methods"; see, for example, [18–21]. These methods focus on, for example, elliptic PDE problems posed on irregular shaped domains. The idea is to embed the irregular domain within a simpler computational domain, such as a rectangular box or sphere. Such an embedding proves advantageous when well-known solution methods (i.e. integral representations and/or fast numerical techniques) are available for the simpler domain. The challenge is then to treat the physical boundary which now arises as an irregular surface inside the computational domain. Similar issues arise when the coefficients defining an elliptic PDE are discontinuous on irregular surfaces within a computational domain. The Immersed Interface Method [22,23] addresses both kinds of problems. Ref. [24] presents recent work on such problems, and it gives a more comprehensive literature review.

In what remains of this introduction, we (1) overview numerical methods for BNS initial data, (2) more fully describe our own first goal, and (3) outline the sections of this paper.

1.1. Overview of numerical methods for BNS initial data

Recent BNS evolutions start with initial data constructed via one of two approaches. The first approach stems from methods introduced by J.W. York, both in its original form [1,2] and in a 1999 reworking partly with H.P. Pfeiffer known as the *conformal thin sandwich (CTS) method* [3,4]. The second approach yields *Meudon data* [25–27], named after its developing group. In principle, it yields a complete solution to Einstein's equations, rather than just initial data.

A number of codes are now capable of generating BNS initial data for the combined matter-Einstein equations, a problem considerably more challenging than the test problems considered here. These codes are reviewed in [28], and the following is based in part on that review. The Princeton-group code described in [29] relies on AMR and the multigrid technique. The BAM code [30] also has a multigrid solver, one relying on nested boxes and Gauss–Seidel relaxation. The recent COCAL code [31] features coordinate patches on which appropriate Green's function expansions are exploited to solve relevant Poisson-type nonlinear equations. The remaining codes, discussed now, are based on nodal spectral methods.

To our knowledge, the first work on spectral methods with GR applications in mind appeared in the proceedings of ICOSAHOM 95 [32].[1] Ultimately, the ideas presented in that volume were developed [25–27] and incorporated into the LORENE code [34] for solving BNS problems. LORENE is a multidomain collocation code based on two nested sets of subdomains, each a family of (at least two) concentric spherical shells with a center-filling ball. One set of subdomains is associated with each star. Although this configuration involves significant subdomain overlap, it is a strikingly simple solution to the nontrivial geometry associated with BNS problems. Using a variant of the self-consistent field method described below, the LORENE code solves the helically symmetric Einstein equations subject to a conformal-flatness assumption.

The Simulating Extreme Spacetimes (SXS) project [35], with key groups at Cornell, Caltech, CITA, and CSU Fullerton (and elsewhere), has also adopted spectral methods to model binary inspiral and merger. Indeed, the Spectral Einstein Code (SpEC) [36] – developed chiefly by L. Kidder, H. Pfeiffer, and M. Scheel – also makes use of multidomain collocation methods. In particular, Pfeiffer's EllipticSolver module [37] within SpEC involves covering the spacetime surrounding two neutron stars (say) with a complicated arrangement of blocks, spherical shells, and cylindrical shells. Coupled elliptic systems of PDE posed on such a domain are then solved using iterative methods (typically GMRES) with a finite-difference preconditioning employed for efficiency and accuracy. For our own work we have made direct use of the domain decomposition used by SpEC's EllipticSolver. Descriptions of BNS initial data generated with SpEC appear in [38–40].

The SGRID code [41,42,28] also relies on nodal spectral methods, along with a novel approach for covering the computational domain developed in particular by Tichy [41]. The approach features coordinate patches surrounding each compact object similar to those introduced by Ansorg [43], rather than the spherical domains used in LORENE and SpEC. The basis functions corresponding to these patches are tensor products of two Chebyshev polynomials and one trigonometric polynomial, that is a double-Chebyshev/Fourier basis.

Insofar as the treatment of stellar surfaces is concerned, the LORENE, SpEC, and SGRID projects employ coordinate mappings to distort (spherical, spherical shell, or patch) subdomains in order that the stellar surface arises precisely as a subdomain interface. Such mapping ameliorates the Gibb's phenomena, although achieving "spectral convergence" for "stiff" equations of state is (in our understanding) still an open problem. Moreover, distortion of the subdomain necessitates regridding, thereby introducing non-trivial Jacobians and changing the bulk operators. As a result, preconditioner recomputation at each iteration stage may be necessary to prevent stalling of the adopted iterative method.

[1] The fundamental reference [33] on "integration preconditioning", the basis for our own work, appears in the same volume.