



A ray-based IPDG method for high-frequency time-domain acoustic wave propagation in inhomogeneous media

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ABSTRACT

The numerical approximation of high-frequency wave propagation in inhomogeneous media is a challenging problem. In particular, computing high-frequency solutions by direct simulations requires several points per wavelength for stability and usually requires many points per wavelength for a satisfactory accuracy. In this paper, we propose a new method for the acoustic wave equation in inhomogeneous media in the time domain to achieve superior accuracy and stability without using a large number of unknowns. The method is based on a discontinuous Galerkin discretization together with carefully chosen basis functions. To obtain the basis functions, we use the idea from geometrical optics and construct the basis functions by using the leading order term in the asymptotic expansion. Also, we use a wavefront tracking method and a dimension reduction procedure to obtain dominant rays in each cell. We show numerically that the accuracy of the numerical solutions computed by our method is significantly higher than that computed by the IPDG method using polynomials. Moreover, the relative errors of our method grow only moderately as the frequency increases.

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1. Introduction

In this paper, we consider the acoustic wave equation in inhomogeneous media given by

$$u_{tt}(x, t) - c^2(x) \Delta u(x, t) = 0, \quad (x, t) \in \mathbb{R}^2 \times [0, T], \quad (1)$$

with initial conditions $u(x, 0) = \sum_{l=1}^L A_l(x) e^{i\omega\phi_l(x)}$ and $u_t(x, 0) = \sum_{l=1}^L i\omega B_l(x) e^{i\omega\phi_l(x)}$, where $c(x) > 0$ is the wave speed in the medium, $T > 0$ is a given time and ω is the frequency of the input. Due to the use of the theory of geometrical optics in the construction of basis functions, we assume that the initial condition $u(x, 0)$ is a superposition of functions in the form $A(x) e^{i\omega\phi(x)}$ for some smooth functions $A(x)$ and $\phi(x)$ independent of ω , and the initial condition $u_t(x, 0)$ is a superposition of functions in the form $i\omega B(x) e^{i\omega\phi(x)}$ where the amplitude function $B(x)$ is also a smooth function independent of the frequency ω . We note that, for more general cases, one can use the technique of micro-local analysis to express the initial conditions as a superposition of functions in the form $A(x) e^{i\omega\phi(x)}$ [33,34].

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1.1. Our work

We consider high-frequency solutions of the problem (1) given by a superposition of wave components:

$$u(x, t) \approx \text{superposition of } \left\{ A_k(x, t) e^{i\omega\phi_k(x, t)} \right\}_{k=1}^N, \quad (2)$$

where $\omega \gg 1$ is the base frequency and ϕ_k are the phase functions that satisfies

$$(\phi_k)_t(x, t) \pm |\nabla\phi_k(x, t)| c(x) = 0.$$

In our method, we assume the following:

- the speed $c(x)$ is smooth in the sense that the variation within each cell of underlying domain partition is small;
- the expansion (2) holds in each cell of an underlying partition of the computational domain, and the number of waves N can vary from cell to cell;
- there is a small number of phases which contribute most to the solution.

In this paper, we consider a modified *wavefront-tracking method* [5,30,35] to capture possible phases in the solution before the actual simulation. Then, we apply a clustering procedure and a dimension reduction procedure to this set of all possible phases and obtain the dominant phases $\phi_k(x)$ in the solution. Using these dominant phases, we define our basis functions as $A_k(x) e^{i\omega\phi_k(x)}$, where A_k are polynomials. We remark that these basis functions are obtained for each cell in the partition of the domain. Finally, we use these basis functions together with an interior penalty discontinuous Galerkin (IPDG) method and obtain the approximate solution by solving the resulting linear system. We note that the basis functions can also be used for other DG discretizations such as [9,10]. We also remark that the wavefront-tracking method can be implemented in a parallel way effortlessly. Moreover, we propose an online/offline scheme to accelerate the simulations for a given medium with many initial conditions, and this situation arises in many practical applications involving inversions. Notice that we will consider smooth media in this paper. This means that the computational mesh is fine enough to capture the variations in the media. For media with more oscillations that cannot be captured by the computational grid, one has to apply some type of multiscale ideas, such as [8,11,16,17]. By using our proposed scheme, we are able to compute the solution of the acoustic wave equation in the high-frequency regime in a very efficient way. Our numerical results show that our scheme is robust on the frequency, i.e. the errors do not grow significantly with the frequency. Furthermore, we compare our scheme with the IPDG scheme that uses the standard polynomial basis. We observe that, with about the same number of unknowns, our scheme performs much better than standard schemes. To the best of our knowledge, our method is the first one that combines ray-based idea and Galerkin method to solve time-dependent wave equations with high frequency solutions.

We note that the ideas and algorithms presented in this paper can be extended naturally to solve the acoustic wave equation in three dimensions, except that the wavefronts become surfaces instead of curves. Therefore we have to represent wavefronts by discrete surfaces, for example, using the method in [6].

1.2. Related works

In literature, there are some works that focus on the closely related problem given by the Helmholtz equation

$$\Delta v(x) + \frac{\omega^2}{c^2(x)} v(x) = 0, \quad (3)$$

with frequency $\omega \gg 1$.

On one hand, due to the Shannon's sampling theorem, the minimum requirement for the number of degrees of freedom of representing the solution is of $\mathcal{O}(\omega^d)$. On the other hand, the pollution effect haunts the standard Galerkin methods using low order polynomials as the basis [2,23], i.e. the L^2 -error grows as ω getting large while keeping ωh fixed. A common strategy for solving this problem is to incorporate oscillatory functions into the basis for the Galerkin methods. This approach significantly reduces the number of degrees of freedom required for accurately representing the solution.

Mainly, there are two types of methods using this strategy. One type of methods does not assume knowledge of the propagation directions of the wave-field. Instead, they incorporate analytic or approximate solutions with predefined directions into the basis. For example, the generalized finite element method [1,31] uses basis function given by the product of plane waves with uniformly-spaced directions and finite element basis functions. The Trefftz methods use local solutions of the Helmholtz equation as the basis functions [12,21]. In particular, a common choice of Trefftz basis for a homogeneous medium is the plane waves, for example, as used in the plane wave discontinuous Galerkin method [18,20,22] and the discontinuous enrichment method [15]. For an inhomogeneous medium, some recent papers have proposed methods to construct the basis consisting of approximate local solutions of the form $e^{P(x)}$ with a complex polynomial P [24–26].

Another type of methods relies on an asymptotic approximation, i.e. the WKB ansatz. This ansatz assumes that a high-frequency solution of the Helmholtz equation (3) can be locally approximated by a superposition of wave components,

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