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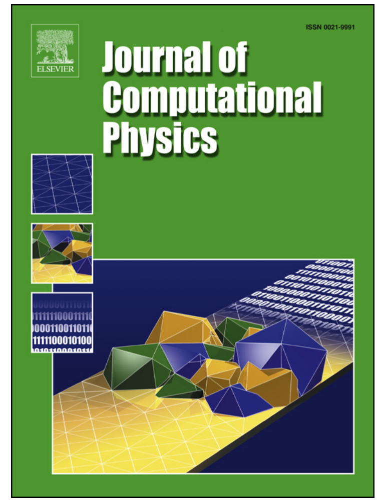

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Parametrization of the cumulant lattice Boltzmann method for fourth order accurate diffusion Part I: derivation and validation[1]

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Abstract

The cumulant lattice Boltzmann method offers a set off free relaxation parameters that do not influence the result at leading order but can be used to influence the leading error. Using Taylor expansion we derive exact functional relationships for the elimination of the linearized leading error of the method. The diffusion term in the Navier-Stokes equation becomes fourth order accurate for small enough viscosity with these parameters. The result is general and does not depend on the flow. The analytical solution is tested against Taylor-Green vortex flow and shear wave flow and fourth order accuracy for the diffusion is observed.

Keywords: lattice Boltzmann, cumulants, quartic parameters, multiple relaxation times, fourth order

1. Introduction

Solving the Navier-Stokes equation at high Reynolds numbers is a problem of significant industrial importance. At the same time it is among the computationally most demanding tasks. The progress in computational hardware, while being impressive, is still insufficient for meeting the demand. Therefore, researchers seek ways to improve the efficiency of their computational methods. In this regard, measures that increase the convergence order of a numerical method are among the most interesting as they promise to make the best use of computational resources. This is especially so if they are combined with large computational resources and applied at high resolution. On the other hand, increasing the convergence order of a numerical method usually involves a steep increase in the complexity of the model. In the case of stencil based methods, larger neighborhoods are usually required to increase the convergence order. In this paper we evaluate the possibility to improve the convergence order of the lattice Boltzmann method without increasing the size of the stencil and at minimal computational overhead. We derive a parametrization that improves the convergence for the diffusion term in the Navier-Stokes equation modeled by the cumulant lattice Boltzmann method to fourth order for sufficiently low viscosity.

It is known for some time [1] that lattice Boltzmann schemes with multiple relaxation times (MRT) [2, 3, 4] offer a parametrization where the numerical error, either for the Navier-Stokes, the Stokes or the advection diffusion equation, for steady solutions at fixed resolution becomes a function of a certain combination of odd and even relaxation rates [5]. Keeping this combination fixed makes the error independent of the transport coefficients, such as viscosity or heat conductivity. This principle has first been applied by Ginzburg and Adler [6] to investigate the apparent dependence of boundary locations for simple boundary conditions. Later, more advanced boundary conditions were derived that apply the parametrization to obtain very accurate solutions [7, 8, 9, 5].

From the finding that the errors depend only on a specific, so-called magic parameter [1] naturally arises the question whether this dependence can be made to zero at any predefined asymptotic order, i.e. whether it is possible to turn a second order numerical scheme, such as the lattice Boltzmann method, into a fourth order accurate scheme

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