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Memory-optimized shift operator alternating direction implicit finite difference time domain method for plasma

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ABSTRACT

Through introducing the alternating direction implicit (ADI) technique and the memory-optimized algorithm to the shift operator (SO) finite difference time domain (FDTD) method, the memory-optimized SO-ADI FDTD for nonmagnetized collisional plasma is proposed and the corresponding formulae of the proposed method for programming are deduced. In order to further the computational efficiency, the iteration method rather than Gauss elimination method is employed to solve the equation set in the derivation of the formulae. Complicated transformations and convolutions are avoided in the proposed method compared with the Z transforms (ZT) ADI FDTD method and the piecewise linear JE recursive convolution (PLJERC) ADI FDTD method. The numerical dispersion of the SO-ADI FDTD method with different plasma frequencies and electron collision frequencies is analyzed and the appropriate ratio of grid size to the minimum wavelength is given. The accuracy of the proposed method is validated by the reflection coefficient test on a nonmagnetized collisional plasma sheet. The testing results show that the proposed method is advantageous for improving computational efficiency and saving computer memory. The reflection coefficient of a perfect electric conductor (PEC) sheet covered by multilayer plasma and the RCS of the objects coated by plasma are calculated by the proposed method and the simulation results are analyzed.

1. Introduction

The finite difference time domain (FDTD) method is one of the most popular numerical methods for modeling the electromagnetic problems because it is easier and more effective compared with other numerical methods and analytical ones [1,2]. Although the FDTD method has been about 40 years since it was first proposed by Yee, its popularity keeps on growing among investigators as time to solution and required memory continue to decline. In addition, numerous extensions and improvements are continuously being presented to further the application fields of the FDTD method. Over the recent years, the FDTD method has been successfully extended to solve the problems of dispersive and complicated materials due to its ability of acquiring wideband results with transient excitation. Investigators have proposed several effective schemes for the FDTD method to deal with dispersive mediums such as plasma.

The recursive convolution (RC) technique was first added to the FDTD method for modeling plasma mediums by Luebbers et al. in 1990 [3]. Thereafter, the RC FDTD method are continuously improved aimed at obtaining higher accuracy and several improved RC FDTD methods are published including piecewise linear recursive convolution (PLRC) FDTD [4],

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trapezoidal recursive convolution (TRC) FDTD [5], JE convolution (JEC) FDTD [6], piecewise linear current density recursive convolution (PLCDRC) FDTD [7] and so on. Although these FDTD methods can tackle dispersive mediums effectively, complicated convolutions are involved in these methods, which may make formulae derivation and programming difficult and lower computational efficiency. Kashiwa and cooperators put forward the auxiliary differential equation (ADE) scheme to deal with Debye and Lorentz mediums [8]. The shortages of the ADE FDTD method are that it needs many additional real variables and many complex exponential variables are included in iteration formulae. The Z transforms (ZT) FDTD method whose dispersive formulation is on the basis of Z transforms was published in Ref. [9] by Sullivan. However, there are many complex transformations from time domain and frequency domain to Z domain in the formulae of the ZT FDTD method. The constitutive relationship of plasma in frequency domain can be written as rational fractional functions, thus the constitutive relationship between electric flux density **D** and electric field vector **E** in time domain can be transformed to discrete time domain through a shift operator (SO). Hence, the SO approach combined with the FDTD method [10,11] is used as an alternative for the solution of dispersive plasma mediums. Compared with the aforementioned methods, the advantages of the SO FDTD method are that there are no complicated convolutions and transformations in its formulae and it is clear in its conception and easy to deduce its formulations. A lot of investigations to the electromagnetic characteristics of dispersive plasma mediums have been done through the SO FDTD method [12–14].

It is commonly known that the FDTD method is a time-consuming method for the reason that its maximum time step size for simulation is limited by the Courant–Friedrichs–Lewy (CFL) condition. Thus, the computational efficiency is constrained, especially for dealing with fine geometry. The alternating direction implicit (ADI) FDTD method which is unconditionally stable [15] is developed to improve computational efficiency for the sake of its overcoming the CFL limit. The ADI scheme has been introduced to several dispersive FDTD techniques because of its high efficiency [16,17]. Nevertheless, there are no reports about adding the ADI scheme to the SO FDTD method to make the SO FDTD method free of CFL limit and improve its computational efficiency. The comparatively low computational efficiency has restricted its wider applications to dispersive medium. Moreover, numerous storage arrays are required in the SO FDTD method because the former field vectors need to be stored in the derivation of the constitutive relationship formula between **D** and **E**. Hence, programming the SO FDTD method occupies much computer memory, which may lower the computational speed and efficiency.

In this paper, the memory-optimized SO-ADI FDTD method for the analysis of nonmagnetized collisional plasma is presented. Through combining the SO technique and the ADI technique with the FDTD method, the proposed FDTD method avoids complicated convolutions and transformations and meanwhile has high computational efficiency compared with the PLJERC-ADI FDTD method and the ZT-ADI FDTD method. The memory-optimized algorithm introduced in the proposed method reduces the storage arrays requirement, which reduces the memory used by the proposed method. Using the iteration method to solve the equation set during the iteration procedure, the efficiency of the proposed method is further improved. To analyze the accuracy of the proposed method, the numerical dispersion error of the SO-ADI FDTD method under different plasma parameters is calculated. The validity and accuracy of the memory-optimized SO-ADI FDTD method is confirmed through comparing the reflection and transmission coefficients of a nonmagnetized collisional plasma sheet obtained by the proposed method with those obtained by the analytical method and other numerical methods. The electromagnetic wave propagation characteristics of a nonuniform multilayer plasma sheet and the scattering characteristics of the objects coated by plasma are investigated via the presented method and the simulation results are discussed.

2. Methodology

For an isotropic nonmagnetized plasma medium with collision, the time-dependent Maxwell's equations and constitutive equations for plasma are given by [18]

$$\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H} \tag{1}$$

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu_0}\nabla \times \mathbf{E} \tag{2}$$

$$\mathbf{D}(\omega) = \varepsilon_0 \varepsilon_r(\omega)\mathbf{E}(\omega) \tag{3}$$

$$\varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + v_{en}^2} - \mathrm{j}\frac{v_{en}}{\omega}\frac{\omega_p^2}{\omega^2 + v_{en}^2} \tag{4}$$

where **D** is electric flux density, **H** is magnetic field, **E** is electric field. ε_0 and μ_0 are permittivity and permeability in vacuum, respectively, $\varepsilon_r(\omega)$ is relative permittivity, v_{en} is electron collision frequency, ω_p is plasma frequency and ω is the frequency of incident electromagnetic wave.

In the SO-ADI FDTD method, the grid configuration of electric field and magnetic field is identical to that of the conventional FDTD method, but one time step of the conventional FDTD method is split into two sub time steps in the SO-ADI FDTD method. Employing the central difference to discretize the x component of Eq. (1), the first sub iteration equation of D_x from the nth to the $(n+1/2)$th time step is

$$D_x|_{i+1/2,j,k}^{n+1/2} = D_x|_{i+1/2,j,k}^{n} + \frac{\Delta t}{2}\left(\frac{H_z|_{i+1/2,j+1/2,k}^{n} - H_z|_{i+1/2,j-1/2,k}^{n}}{\Delta y} - \frac{H_y|_{i+1/2,j,k+1/2}^{n+1/2} - H_y|_{i+1/2,j,k-1/2}^{n+1/2}}{\Delta z}\right) \tag{5}$$