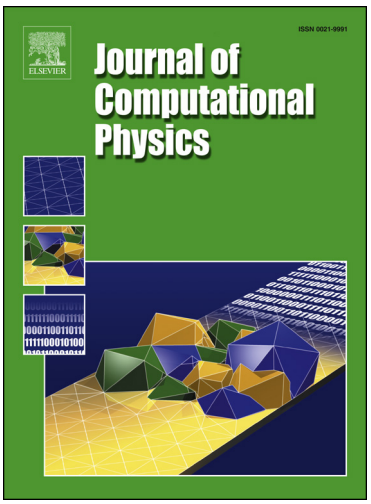
Accepted Manuscript



A sixth-order finite volume scheme for the steady-state incompressible Stokes equations on staggered unstructured meshes

Ricardo Costa, Stéphane Clain, Gaspar J. Machado

PII: S0021-9991(17)30555-7
DOI: http://dx.doi.org/10.1016/j.jcp.2017.07.047
Reference: YJCPH 7494

To appear in: *Journal of Computational Physics*

Received date: 5 April 2016
Revised date: 24 July 2017
Accepted date: 25 July 2017

Please cite this article in press as: R. Costa et al., A sixth-order finite volume scheme for the steady-state incompressible Stokes equations on staggered unstructured meshes, *J. Comput. Phys.* (2017), http://dx.doi.org/10.1016/j.jcp.2017.07.047

A sixth-order finite volume scheme for the steady-state incompressible Stokes equations on staggered unstructured meshes

Ricardo Costa[a], Stéphane Clain[b], Gaspar J. Machado[b]

[a]*Institut de Mathématiques de Toulouse, Université Paul Sabatier, 31062 Toulouse, France*
e-mail: rcosta@math.univ-toulouse.fr
[b]*Centre of Mathematics, University of Minho, Campus of Azurém, 4800-058 Guimarães, Portugal*
e-mail: {clain,gjm}@math.uminho.pt

Abstract

We propose a new sixth-order finite volume scheme to solve the bidimensional linear steady-state Stokes problem on staggered unstructured meshes and complex geometries. The method is based on several classes of polynomial reconstructions to accurately evaluate the diffusive fluxes, the pressure gradient, and the velocity divergence. The main difficulty is to handle the div-grad duality to avoid numerical locking and oscillations. A new preconditioning technique based on the construction of a pseudo-inverse matrix is also proposed to dramatically reduce the computational effort. Several numerical simulations are carried out to highlight the performance of the new method.

Keywords: Stokes equations, incompressible fluid, finite volume, high-order scheme, preconditioning

1. Introduction

The Stokes problem together with the Darcy system are two classical prototype models of mixed problems where the pressure function derives from the divergence-free velocity constraint. The finite volume method turns to be a natural framework to design built-in conservation schemes since the pioneer book of Patankar [39] and we refer to the textbook of Ferziger and Peric [17] for an overview of the finite volume for the Navier-Stokes equations. A large range of schemes has been developed to provide both accurate and stable solutions where one can distinguish different kinds of approach namely staggered or collocated discretizations, structured grids or unstructured meshes for complex geometries, and coupled or segregated velocity and pressure leading to a saddle point problem or a projection method in the divergence-free space (see the introductions of [22, 44] for a short overview). Another fundamental challenge concerns the preconditioning of the linear system deriving from the space discretization.

Second-order methods are a standard in industry for the computation of incompressible flow and in commercial software development. There exists an important literature and books on the subject using the finite difference [8, 41], the finite element [43, 52, 7, 23, 54], or the finite volume approach [39, 20, 53, 49, 15, 5]. However, there