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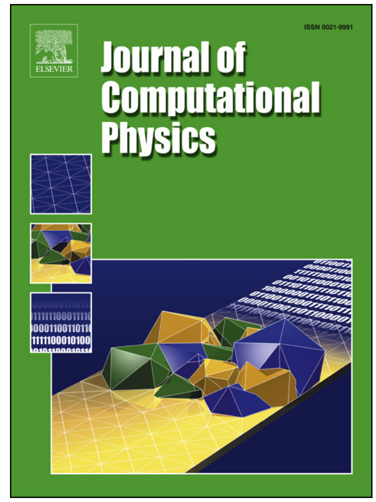

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Muhammad Mohebujjaman, Leo G. Rebholz, Xuping Xie, Traian Iliescu

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Energy Balance and Mass Conservation in Reduced Order Models of Fluid Flows

Muhammad Mohebujjaman[a], Leo G. Rebholz[a,*], Xuping Xie[b], Traian Iliescu[b],

[a] Department of Mathematical Sciences, Clemson University, Clemson, SC 29634, U.S.A.
[b] Department of Mathematics, Virginia Tech, Blacksburg, VA 24061, U.S.A.

Abstract

In this paper, we investigate theoretically and computationally the conservation properties of reduced order models (ROMs) for fluid flows. Specifically, we investigate whether the ROMs satisfy the same (or similar) energy balance and mass conservation as those satisfied by the Navier-Stokes equations. All of our theoretical findings are illustrated and tested in numerical simulations of a 2D flow past a circular cylinder at a Reynolds number $Re = 100$.

First, we investigate the ROM energy balance. We show that using the snapshot average for the centering trajectory (which is a popular treatment of nonhomogeneous boundary conditions in ROMs) yields an incorrect energy balance. Then, we propose a new approach, in which we replace the snapshot average with the Stokes extension. Theoretically, the Stokes extension produces an accurate energy balance. Numerically, the Stokes extension yields more accurate results than the standard snapshot average, especially for longer time intervals.

Our second contribution centers around ROM mass conservation. We consider ROMs created using two types of finite elements: the standard Taylor-Hood (TH) element, which satisfies the mass conservation weakly, and the Scott-Vogelius (SV) element, which satisfies the mass conservation pointwise. Theoretically, the error estimates for the SV-ROM are sharper than those for the TH-ROM. Numerically, the SV-ROM yields significantly more accurate results, especially for coarser meshes and longer time intervals.

Keywords: Reduced order model, proper orthogonal decomposition, energy balance, mass conservation.

1. Introduction

Reduced order models (ROMs) have been used in numerous scientific and engineering applications, and have been particularly successful as surrogate models for structure dominated fluid flows (see e.g. the nice review of Lassila et al in [1] and references therein). The ROM strategy is straightforward: In an offline stage, snapshots from a fine resolution numerical simulation of the

*corresponding author

Email addresses: mmohebu@clemson.edu (Muhammad Mohebujjaman), rebholz@clemson.edu (Leo G. Rebholz), xupingxy@vt.edu (Xuping Xie), iliescu@vt.edu (Traian Iliescu)

URL: https://mthsc.clemson.edu/directory/viewperson.py?personid=358 (Muhammad Mohebujjaman), http://www.math.clemson.edu/~rebholz (Leo G. Rebholz), http://www.math.vt.edu/people/xupingxy (Xuping Xie), http://www.math.vt.edu/people/iliescu (Traian Iliescu)