Journal of Computational Physics 346 (2017) 295–317

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Axisymmetric charge-conservative electromagnetic particle simulation algorithm on unstructured grids: Application to microwave vacuum electronic devices

Dong-Yeop Na [a,*], Yuri A. Omelchenko [b], Haksu Moon [c], Ben-Hur V. Borges [d], Fernando L. Teixeira [a]

[a] ElectroScience Laboratory and Department of Electrical and Computer Engineering, The Ohio State University, Columbus, OH 43212, USA
[b] Trinum Research Inc., San Diego, CA 92126, USA
[c] Intel Corporation, Hillsboro, OR 97124, USA
[d] Department of Electrical and Computer Engineering, University of Sao Paulo, Sao Carlos, SP 13566-590, Brazil

ARTICLE INFO

Article history:
Received 5 January 2017
Received in revised form 21 April 2017
Accepted 7 June 2017
Available online 15 June 2017

Keywords:
Electromagnetic simulation
Particle-in-cell
Unstructured meshes
Vacuum electronic devices
Backward-wave oscillator
Cerenkov radiation

ABSTRACT

We present a charge-conservative electromagnetic particle-in-cell (EM-PIC) algorithm optimized for the analysis of vacuum electronic devices (VEDs) with cylindrical symmetry (axisymmetry). We exploit the axisymmetry present in the device geometry, fields, and sources to reduce the dimensionality of the problem from 3D to 2D. Further, we employ 'transformation optics' principles to map the original problem in polar coordinates with metric tensor $\mathrm{diag}(1, \rho^2, 1)$ to an equivalent problem on a Cartesian metric tensor $\mathrm{diag}(1, 1, 1)$ with an effective (artificial) inhomogeneous medium introduced. The resulting problem in the meridian (ρz) plane is discretized using an unstructured 2D mesh considering TE^{ϕ}-polarized fields. Electromagnetic field and source (node-based charges and edge-based currents) variables are expressed as differential forms of various degrees, and discretized using Whitney forms. Using leapfrog time integration, we obtain a mixed $\mathcal{E} - \mathcal{B}$ finite-element time-domain scheme for the full-discrete Maxwell's equations. We achieve a local and explicit time update for the field equations by employing the sparse approximate inverse (SPAI) algorithm. Interpolating field values to particles' positions for solving Newton–Lorentz equations of motion is also done via Whitney forms. Particles are advanced using the Boris algorithm with relativistic correction. A recently introduced charge-conserving scatter scheme tailored for 2D unstructured grids is used in the scatter step. The algorithm is validated considering cylindrical cavity and space-charge-limited cylindrical diode problems. We use the algorithm to investigate the physical performance of VEDs designed to harness particle bunching effects arising from the coherent (resonance) Cerenkov electron beam interactions within micro-machined slow wave structures.

1. Introduction

Historically the need for high-power electromagnetic (EM) radiation sources in the gigahertz and terahertz frequency ranges has triggered significant technical advances in vacuum electronic devices (VEDs) [1–5], such as the gyrotron, free

* Corresponding author.
E-mail addresses: na.94@osu.edu (D.-Y. Na), omelche@gmail.com (Y.A. Omelchenko), haksu.moon@gmail.com (H. Moon), benhur@sc.usp.br (B.-H.V. Borges), teixeira.5@osu.edu (F.L. Teixeira).

http://dx.doi.org/10.1016/j.jcp.2017.06.016

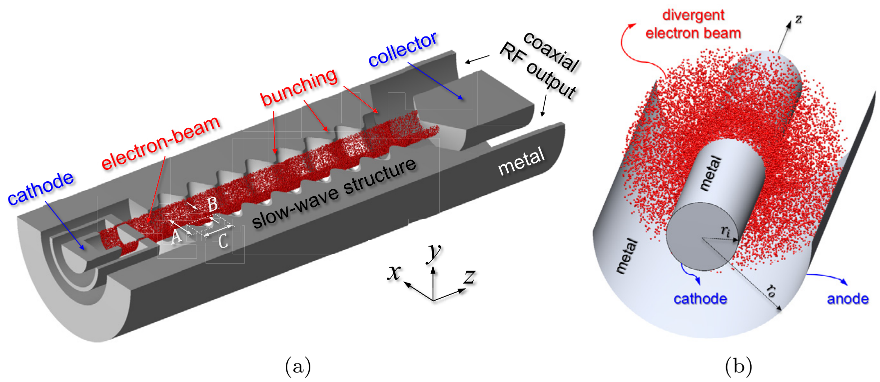

Fig. 1. Schematics of two examples of axisymmetric vacuum electronic devices. (a) Backward-wave oscillator producing bunching effects on an electron beam. Wall ripples are designed to support slow-wave modes in the device. (b) Space-charge-limited cylindrical vacuum diode.

electron Laser, and traveling wave tube (TWT). These devices serve as a basis for a variety of applications in radar and communications systems, plasma heating for fusion, and radio-frequency (RF) accelerators [6,7].

Amplification of RF signals is usually obtained by exploiting resonance Cerenkov interactions between an electron beam and the modal field supported by a slow-wave structure (SWS) [2,8–11]. SWSs are often made by imposing periodic ripples on the conducting wall of cylindrically symmetric waveguides, as illustrated in Fig. 1a, so that the phase velocity of the modal field becomes slower than the speed of light in vacuum due to Bragg scattering [12]. According to the dispersion relations associated with the geometry of SWSs, resultant Cerenkov interactions can amplify forward or backward waves.[1] Similarly to plasma instabilities, the evolution of forward and backward waves can be characterized by *convective* instabilities that grow over time while traveling away from the location of initial disturbance and *absolute* instabilities that propagate a local initial disturbance throughout the whole device volume [2]. Traveling-wave tube amplifiers (TWTA) and backward-wave oscillators (BWO) are two practical examples utilizing convective and absolute instabilities, respectively.

Recent studies have shown that a particular SWS geometry may significantly enhance the system performance of TWTs. For example, nonuniform (locally periodic) ripples used in BWOs may improve mode conversion efficiency [13,14], and tapering ripples may reduce reflections at the output of TWTA and prevent internal oscillations [8,15]. More importantly, smooth device edges are preferred for high-output power applications in order to mitigate pulse shortening, which is a major bottleneck for increasing output powers beyond the gigawatts range [16,17]. This is because extremely strong field singularities, which accumulate on the sharp edges, may create interfering plasmas that terminate the output signal at an earlier time. Sinusoidally corrugated slow wave structures (SCSWS) have been increasingly adopted in many modern high-power BWO systems to combat this problem [18]. In addition, a variety of micro-machining fabrication techniques have been developed to enable better device performances by using much tighter tolerances. These technological advances have allowed the production of devices operating at higher frequencies, including the THz regime.

Computational experiments for VEDs employ electromagnetic particle-in-cell (EM-PIC) algorithms [19–22], which numerically solve the Maxwell–Vlasov equations describing weakly coupled (collision-less) systems, where the collective behavior of charged particles prevails over their binary collisions [19,20,23]. A typical PIC algorithm tracks the temporal evolution of macro-particles seeded in a coarse-grained six-dimensional (6D) phase space.[2] A typical PIC algorithm consists of four basic steps, viz. the field solver, the field gather, the particle push, and the particle charge/current scatter, which are repeated at every time iteration. This provides a self-consistent update of particles and field states in time.

As a field solver, most EM-PIC simulations employ the celebrated Yee's finite-difference time-domain (FDTD) method for regular grids, due to its simplicity. There are a plethora of FDTD-based EM-PIC codes such as `UNIPIC`, `MAGIC`, `TWOQUICK`, `KARAT`, `VORPAL`, and others [24–26]. However, the relatively poor grid-dispersion properties of this algorithm [27] causes spurious numerical Cerenkov radiation [28]. Moreover, in complex geometries such as those of modern VEDs, "staircase" (step-cell) effects present a critical challenge. Using FDTD for an accurate analysis of geometrically complex devices, which typically have curved boundaries or very fine geometrical features, may require excessive mesh refinement and therefore result in a waste of computational resources. Many studies have been done to mitigate the staircasing errors in finite-difference (FD) methods, in particular through using conformal FD discretizations [29,30].

On the other hand, the finite-element time-domain (FETD) method [31,32] fundamentally eliminates the undesired staircase effects since it is naturally based on unstructured (irregular) grids, which can more easily be made conformal to complex geometries and can be augmented by powerful mesh refinement algorithms. Unfortunately, conventional FETD-based PIC algorithms have historically faced numerical challenges that result from a lack of exact charge conservation on unstructured grids. This gives rise to the accumulation of spurious charges which must be removed by applying costly a pos-

1 Along the direction of the group velocity w.r.t. the beam velocity.
2 That is, a finite-size ensemble of physical particles with positions and momenta.