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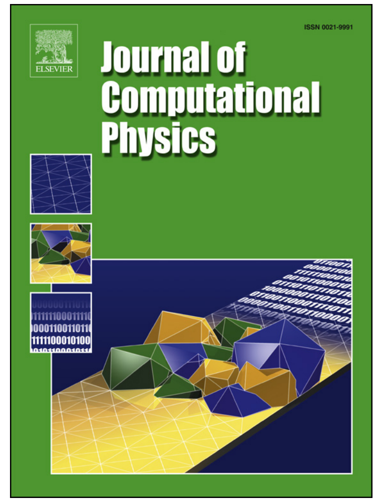

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Fully implicit mixed-hybrid finite-element discretization for general purpose subsurface reservoir simulation

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Abstract

We present a new fully-implicit, mixed-hybrid, finite-element (MHFE) discretization scheme for general-purpose compositional reservoir simulation. The locally conservative scheme solves the coupled momentum and mass balance equations simultaneously, and the fluid system is modeled using a cubic equation-of-state. We introduce a new conservative flux approach for the mass balance equations for this fully-implicit approach. We discuss the nonlinear solution procedure for the proposed approach, and we present extensive numerical tests to demonstrate the convergence and accuracy of the MHFE method using tetrahedral elements. We also compare4 the method to other advanced discretization schemes for unstructured meshes and tensor permeability. Finally, we illustrate the applicability and robustness of the method for highly heterogeneous reservoirs with unstructured grids.

Keywords: Fully implicit, mixed-hybrid finite element, momentum and mass coupling, finite volume, reservoir simulation, unstructured grids, compositional modeling, full tensor.

1. Introduction

Management of subsurface resources, including water aquifers and oil/gas reservoirs relies on numerical reservoir simulation. Flow simulation entails discretization of the equations and constitutive relations that describe multi-component, multiphase flow in subsurface porous formations. The reservoir characterization model (RCM) that serves as input to the flow simulator can be quite complex, both in terms of the grid geometry and the formation properties (e.g., permeability). Each gridblock is assigned material properties and, in most cases, the properties are different in each block. For highly-heterogeneous fields, comprised of many stratigraphic compartments, the number of gridblocks can be extremely large, and the computational domain can be highly unstructured. Therefore, accurate modeling of the local flow dynamics, especially the fluxes between the gridblocks is of great importance. Moreover, the highly nonlinear nature of the coupled conservation equations and constitutive relations make honoring local conservation a fundamental requirement.

Traditionally, a two-point flux approximation (TPFA) technique is employed to approximate the phase flux in the Darcy equation [1]. As the name implies, two points are used to approximate the flux at the interface between the control-volumes (gridblocks). The TPFA scheme requires the mesh of the domain to be structured, such that the interfaces are orthogonal to the interface between control-volumes, and the grid is aligned with the principal directions of any anisotropic material property, e.g., the permeability. There is strong interest in the community to use high-resolution, geometrically complex RCMs that represent the large-scale subsurface formations as accurately as possible. Accommodating three-dimensional unstructured grids with full-tensor properties is a difficult - practically impossible - task using TPFA. Multi-point flux-approximation (MPFA) discretization schemes have been developed to address issues related to nonorthogonal grids and full-tensor heterogeneous permeability [2, 3]. Similar to TPFA schemes, divergence-free conditions are imposed on the control-volume to approximate the flux. MPFA schemes - subject to limitations - allow for using unstructured grids and anisotropic material properties [4]. Several variations of the method have been developed [5, 6] with corresponding convergence studies [7, 8, 9]. In practice, the 'MPFA-O' method is the most widely used. More recently, nonlinear flux approximation schemes have been developed [10, 11, 12].

Another flux approximation scheme for unstructured meshes is the mixed-finite-element method (MFEM), where the mass and momentum equations are coupled and solved simultaneously [13]. This method is locally conservative, and it can accommodate high-order approximations and anisotropic material properties. However, in its original form, MFEM leads to an algebraic system of saddle-point type, whereby the linearized matrix is indefinite. An efficient variant for the method was

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