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Reduced-order prediction of rogue waves in two-dimensional deep-water waves

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A B S T R A C T

We consider the problem of large wave prediction in two-dimensional water waves. Such waves form due to the synergistic effect of dispersive mixing of smaller wave groups and the action of localized nonlinear wave interactions that leads to focusing. Instead of a direct simulation approach, we rely on the decomposition of the wave field into a discrete set of localized wave groups with optimal length scales and amplitudes. Due to the short-term character of the prediction, these wave groups do not interact and therefore their dynamics can be characterized individually. Using direct numerical simulations of the governing envelope equations we precompute the expected maximum elevation for each of those wave groups. The combination of the wave field decomposition algorithm, which provides information about the statistics of the system, and the precomputed map for the expected wave group elevation, which encodes dynamical information, allows (i) for understanding of how the probability of occurrence of rogue waves changes as the spectrum parameters vary, (ii) the computation of a critical length scale characterizing wave groups with high probability of evolving to rogue waves, and (iii) the formulation of a robust and parsimonious reduced-order prediction scheme for large waves. We assess the validity of this scheme in several cases of ocean wave spectra.

1. Introduction

Rogue waves refer to extremely large oceanic surface waves. As a result of their devastating impact on marine systems, such as ships and offshore platforms, rogue waves have been the subject of numerous theoretical, experimental and numerical studies [6,14,32]. Most studies concern the frequency and statistics of rogue wave occurrence for a given sea state (see, e.g., [20,25,26,40,47]). It is, however, often desirable to know, for a given ocean area, *if*, *when* and *where* a rogue wave may occur in the future.

These questions can in principle be addressed by numerically solving the appropriate hydrodynamic equations [7,13,27]. Apart from its high computational cost, this direct approach requires a well-resolved state of the fluid velocity field and its free surface elevation as initial conditions. Thanks to recent developments, real-time and reliable measurement of sea surface elevation is feasible (see e.g. [22,28,29,39,41]). But the well-resolved measurement of fluid velocity field remains out of reach.

An alternative approach for short-term prediction of the wave field is based on numerically solving the so-called envelope equations, which approximate the evolution of the wave envelope to a reasonable accuracy [15,42,48]. While less

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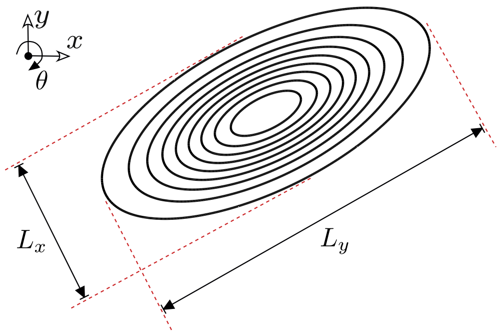

Fig. 1. Schematic view of an elementary wave group (EWG) with a Gaussian profile.

expensive than the full hydrodynamic equations, solving the envelope equations is still computationally formidable for real-time forecast of extreme waves.

As a result, several attempts have been made to devise reliable, reduced-order methods for short-time forecast of water wave evolution. Adcock et al. [1], for instance, approximate nonlinear evolution of localized wave groups with an exact breather-like solution of the linear Schrödinger equation. To account for the nonlinear effects, they allow the parameters of the breather-like solution to vary in time such that particular invariants (energy and the Hamiltonian) of the nonlinear Schrödinger equation (NLS) are preserved over time. Ruban [36] takes a similar approach by substituting a Gaussian ansatz into the Lagrangian functional associated with the NLS equation. The time evolution of the parameters are determined such that the solution satisfies a least-action principle (also see [35]).

The resulting wave groups from Adcock et al. [1] and Ruban [35] do not necessarily satisfy the underlying envelope equation (i.e., the NLS equation). Furthermore, the reduced-order method of Ruban [35] relies heavily on the Lagrangian formulation of the NLS equation. As such, it is not immediately applicable to the more realistic, higher-order envelope equations, such as the modified NLS (MNLS) equation of Dysthe [15], whose Lagrangian formulation is unavailable (see [23] and [12] for the Hamiltonian formulation of the MNLS equation).

To avoid these drawbacks, Cousins and Sapsis [8,10] take an intermediate approach. They also consider the evolution of parametric wave groups but allow the wave group to evolve under the full nonlinear evolution equation by imposing energy conservation [9]. The analysis resulted in a reduced-order set of nonlinear equations that captures the nonlinear dynamics of wave groups and most critically their transition from defocussing to focusing. This reduced-order model which represents information for the dynamics of the wave groups is combined with a probabilistic analysis of the possible wave groups that can form stochastically for a given wave spectrum [10]. Note that stochasticity is inevitably introduced due to the 'mixing' between harmonics that propagate with different speeds due to dispersion. The resulted schemes provide a parsimonious and robust prediction scheme for unidirectional water waves.

The main purpose of the present paper is to extend the framework of Cousins and Sapsis [10] from their unidirectional context to multidirectional water waves. Several new challenges arise in this context that are absent in the unidirectional case. In the following section we review these challenges, summarize our framework and state the assumptions under which this reduced-order framework is applicable.

1.1. Summary of the framework

We seek to approximate the future spatiotemporal maximum wave height of a measured wave field by decomposing the field as the superposition of wave groups with simple shapes. The evolution of the simple wave groups are precomputed and stored, so that the prediction reduces essentially to an interpolation from an existing data set. This reduced-order approach can be divided into the following steps:

I. Evolution of elementary wave groups.
II. Decomposition of random wave fields.
III. Prediction of amplitude growth.

Step I. We consider spatially localized simple wave groups that can be expressed analytically and refer to them as *elementary wave groups* (EWG). In this paper we will use EWG with a Gaussian profile. One can alternatively use other shapes such as the secant hyperbolic used in [10]. The key requirement is that the EWG must be completely determined with only a few parameters. A Gaussian wave group, for instance, is determined by its amplitude (A_0), its longitudinal and transverse widths (L_x and L_y) and its orientation (θ) with respect to a global reference frame (see Fig. 1). Working with the Gaussian is also convenient since its derivatives with respect to parameters and variables take a simple form.

For a realistic range of these parameters, we evolve the corresponding elementary wave groups for T time units by numerically solving an appropriate wave envelope equation (see Section 2). We record the spatiotemporal maximum amplitude A_{max} that each EWG reaches over the time interval $[0, T]$. This step is computationally expensive but is carried out