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# A new high-order finite volume method for 3D elastic wave simulation on unstructured meshes

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ABSTRACT

In this paper, we proposed a new efficient high-order finite volume method for 3D elastic wave simulation on unstructured tetrahedral meshes. With the relative coarse tetrahedral meshes, we make subdivision in each tetrahedron to generate a stencil for the high-order polynomial reconstruction. The subdivision algorithm guarantees the number of subelements is greater than the degrees of freedom of a complete polynomial. We perform the reconstruction on this stencil by using cell-averaged quantities based on the hierarchical orthonormal basis functions. Unlike the traditional high-order finite volume method, our new method has a very local property like DG and can be written as an inner-split computational scheme which is beneficial to reducing computational amount. Moreover, the stencil in our method is easy to generate for all tetrahedrons especially in the three-dimensional case. The resulting reconstruction matrix is invertible and remains unchanged for all tetrahedrons and thus it can be pre-computed and stored before time evolution. These special advantages facilitate the parallelization and high-order computations. We show convergence results obtained with the proposed method up to fifth order accuracy in space. The high-order accuracy in time is obtained by the Runge–Kutta method. Comparisons between numerical and analytic solutions show the proposed method can provide accurate wavefield information. Numerical simulation for a realistic model with complex topography demonstrates the effectiveness and potential applications of our method. Though the method is proposed based on the 3D elastic wave equation, it can be extended to other linear hyperbolic system.



## 1. Introduction

Wave simulation based on wave equations has important applications in geophysics. For example, wave propagation can be applied to detect the structures of oil and gas. Thus an efficient and accurate simulation technique for wave equations is important. There are many numerical methods to solve wave equations, for example, the finite difference (FV) method [36,51], the pseudo-spectral (PS) method [19,29,59], the finite element (FE) method [3,8,60], the spectral ele-

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ment (SE) method [30,31], the discontinuous Galerkin (DG) method [6,15,25] and the finite volume (FV) method [16,17]. Each numerical method has its own inherent advantages and disadvantages. First we give a brief review of these methods.

The FD method is efficient and relatively easy to implement, but the inherent restriction of using regular meshes limits its application to complex topography. The PS method can be viewed as the limit of FD with infinite order of accuracy in space and it computes the spatial derivatives by using the fast Fourier transform (FFT). However, large amount of forward and inverse FFT reduces its computational efficiency.

The FE method is widely used in various problems including wave simulation. With triangle elements in 2D or tetrahedral elements in 3D, this method is suited to model complex topography. However, a global mass matrix is required to be inverted at each time-step which requires large computational cost. In order to overcome the problem, the mass-lumping technique which leads to a diagonal mass matrix is developed [8,9,37]. The SE method overcomes the problem by using Gauss–Labatto quadrature rules [10]. It is originally introduced in the computational fluid mechanics [38] and has been well applied in the problems of wave propagation in geophysics [30–32,39,43].

The DG method is a high-order conservative method which was first proposed for solving neutron transport equation [42]. Combined with a time-integration method named Arbitrary high-order DERivatives (ADER), the DG method has been widely applied to wave simulation in geophysics [15,18,25,27,40,41,56,57]. In contrast to FE, the solution of DG can be discontinuous on the interface of elements and the mass matrix is local rather than global, which especially facilitates parallelization and high-order schemes are relatively easy to establish. However, the DG method needs to compute high-order surface and volume integrals, which can be expensive to compute. For example, a $k$-th order DG scheme requires a $2k$-th order quadrature formula for the surface integrals, and $(2k-1)$-th order for the volume integrals.

The FV method is very popular in solving linear hyperbolic equations [16,17,33]. It can be extended to high accuracy on unstructured grids using a high-order polynomial reconstruction. In the FV method, the discrete values are approximations of cell averages, and then it consists of two steps: the reconstruction and flux calculation. By the reconstruction step, local values are interpolated from the cell-averaged values. The FV method attains spatial high accuracy by a high-order polynomial reconstruction while the discretization in time is usually performed by Runge–Kutta (RK) schemes. In order to improve efficiency of RK schemes as the Butcher barrier [5] when the time accuracy is higher than four, the ADER approach can be exploited for time-integration. The idea of ADER was first proposed by Toro et al. [50] and it has been combined with the DG and FV methods successfully. The resulting ADER-DG and ADER-FV methods are well applied to linear systems including wave equations on unstructured meshes [15–17,25]. Recently, a high-order FV method called the spectral finite volume (SFV) method is developed on unstructured grids [34,48,52–55]. The SFV method is so called to achieve high-order accuracy in an efficient manner similar to the SE method. A detail comparison between DG and SFV may refer to the references [47,58].

In order to simulate wave propagation on unstructured meshes efficiently, the FV method is a good choice due to its high computational efficiency and good adaptability to complex geometry. The aim of this paper is to illustrate a new efficient FV method for 3D elastic wave simulation on unstructured meshes. We will incorporate some nice features from the DG and FV methods [15–18,24,25,27] and the SFV method [34,48,52–55]. In our method, the computational domain is first meshed with relative coarse tetrahedrons. Then, each tetrahedron is further divided as a collection of finer tetrahedral subelements to form a stencil. The high-order polynomial reconstruction is performed on this stencil by using local cell-averaged values on the finer elements. Our method can be considered as the extension of the high-order FV work of Dumbser et al. [18]. Our method has three main advantages. The first one is that the reconstruction matrix on all coarse tetrahedrons remains unchanged and it can be pre-computed and stored before time evolution. The second one is that, by using a suitable number of finer tetrahedrons, we obtain an over-determined reconstruction system which has very local property and this fact also enhances the parallelization of our algorithm. The third one is that our method can be written as an inner-split computational scheme which is beneficial to reducing computational cost. We will present numerical results to show the performance of our method. We remark that the new method can be applied to other linear hyperbolic system without essential difficulty.

The rest of this paper is organized as follows. In Section 2, the theoretical method is described in detail. In Section 3, numerical results are given to illustrate the effectiveness of our method. Finally, conclusions are drawn in Section 4.

## 2. Theoretical method

### 2.1. The governing equation

The 3D elastic wave equation with external sources in velocity–stress formulation can be written as the following system [15,36]