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Overcoming numerical shockwave anomalies using energy balanced numerical schemes. Application to the Shallow Water Equations with discontinuous topography

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ABSTRACT

When designing a numerical scheme for the resolution of conservation laws, the selection of a particular source term discretization (STD) may seem irrelevant whenever it ensures convergence with mesh refinement, but it has a decisive impact on the solution. In the framework of the Shallow Water Equations (SWE), well-balanced STD based on quiescent equilibrium are unable to converge to physically based solutions, which can be constructed considering energy arguments. Energy based discretizations can be designed assuming dissipation or conservation, but in any case, the STD procedure required should not be merely based on ad hoc approximations. The STD proposed in this work is derived from the Generalized Hugoniot Locus obtained from the Generalized Rankine Hugoniot conditions and the Integral Curve across the contact wave associated to the bed step. In any case, the STD must allow energy-dissipative solutions: steady and unsteady hydraulic jumps, for which some numerical anomalies have been documented in the literature. These anomalies are the incorrect positioning of steady jumps and the presence of a spurious spike of discharge inside the cell containing the jump. The former issue can be addressed by proposing a modification of the energy-conservative STD that ensures a correct dissipation rate across the hydraulic jump, whereas the latter is of greater complexity and cannot be fixed by simply choosing a suitable STD, as there are more variables involved. The problem concerning the spike of discharge is a well-known problem in the scientific community, also known as slowly-moving shock anomaly, it is produced by a nonlinearity of the Hugoniot locus connecting the states at both sides of the jump. However, it seems that this issue is more a feature than a problem when considering steady solutions of the SWE containing hydraulic jumps. The presence of the spurious spike in the discharge has been taken for granted and has become a feature of the solution. Even though it does not disturb the rest of the solution in steady cases, when considering transient cases it produces a very undesirable shedding of spurious oscillations downstream that should be circumvented. Based on spike-reducing techniques (originally designed for homogeneous Euler equations) that propose the construction of interpolated fluxes in the untrustworthy regions, we design a novel Roe-type scheme for the SWE with discontinuous topography that reduces the presence of the aforementioned spurious spike. The resulting spike-reducing method in combination with the proposed STD ensures an accurate positioning of steady jumps,

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provides convergence with mesh refinement, which was not possible for previous methods that cannot avoid the spike.

1. Introduction

There is a wide variety of physical problems modeled by non-homogeneous hyperbolic systems of conservation laws that are dominated by source terms. For such problems, the treatment of the source terms when designing a numerical scheme is of utmost importance in order to provide realistic and physically feasible solutions. Depending on the nature of the source term, different numerical techniques may be required. In this work, we focus on a certain type of source term, called geometric source term, present in many physical one-dimensional (1D) problems. This kind of source makes the conserved quantities account for the variation in space of a geometric variable, which is provided in the problem. Examples of mathematical models including geometric source terms are, for instance, the SWE with discontinuous topography, which is the object of study in the present work, the 1D Euler equations in a duct of variable cross section [1] and 1D flow in collapsible vessels [2].

Most popular methods for the resolution of homogeneous hyperbolic problems are within the framework of finite volume Godunov's numerical schemes [3], which aim to provide a numerical solution to the problem by means of a prior discretization of the domain into volume cells and integration of the information and governing equations inside these cells. After integration, simple algebraic evolution equations for the conserved variables, that depend upon the same variables at a previous time step and the fluxes at cell interfaces, arise. The keystone in Godunov's schemes is the computation of the numerical fluxes at cell interfaces, which is carried out by means of the resolution of the so-called Riemann Problems (RPs). RPs are initial value problems defined at cell interfaces, whose initial data is piecewise constant data given by the cell-averaged variables at each side of the discontinuity. They may be regarded as first order approach to the more general Cauchy problem [4].

When dealing with geometric source terms, it is necessary to account for the jump of the geometric quantity across cell interfaces when defining numerical fluxes at cell interfaces. To this end, augmented solvers were introduced [5–7]. When using augmented solvers, the source term is accounted for in the solution of the RP as an extra stationary wave at the interface. Due to the presence of the new wave, two solutions appear now at each side of the initial discontinuity instead of having a single homogeneous solution. Augmented versions of the traditional Roe [8] (ARoe) and HLLC [9,10] solvers were presented by Murillo in [11] and [12] respectively. An extense review of the ARoe method can be found in [13].

If examining the system of equations in the so-called non-conservative form, the contribution of the source term is modeled as an additional stationary wave at the interface, which allows to include the effect of the source term in the eigenstructure of the system. This way, it can be noticed that the presence of a jump in the geometric variable gives rise to a contact wave and furthermore, that Riemann invariants are not necessarily conserved across such a wave, as pointed out by Rosatti et al. [14]. This issue will be recalled when designing the numerical scheme.

In the early stages of the design of numerical schemes for hyperbolic problems with source terms, the main effort was put on how to modify the original schemes, initially designed for homogeneous equations, so that they maintain the discrete equilibrium between fluxes and source term under steady state. When considering realistic applications, such goal was translated into the preservation of physical steady situations of quiescent equilibrium. For instance, in the framework of the SWE, the preservation of the steadiness of the solution for still water at rest. Numerical schemes satisfying this property were called well-balanced schemes [15–19].

When considering steady states with moving water over a irregular bed profile, the preservation of the C-property (exact conservation property) [16] is also of utmost importance in order to provide an exact equilibrium between fluxes and source terms. Numerical methods preserving the C-property are able to ensure a uniform discharge under steady conditions and can be constructed using flux-type definitions of the source terms [20,6].

We can still take the well-balanced and C-property a step further by considering the conservation of the discrete specific mechanical energy in the scheme, enhancing in this way the performance of the numerical method. When friction is not considered in the SWE, mechanical energy is conserved under steady conditions in absence of hydraulic jumps. Such idea of energy conservation can be integrated in the numerical scheme, allowing the extension of well-balanced methods to exactly well-balanced methods [21–26], hereafter referred to as E-schemes. Numerical methods defined as E-schemes will always satisfy the energy conservation property in the discrete level, hereafter referred to as E-property. Arbitrary order augmented Roe and HLL schemes preserving the E-property, called AR-ADER and HLLS-ADER E-schemes respectively, were presented by the authors of this work in [27,28] and applied to the SWE. As a result of preserving the E-property, the aforementioned schemes were able to provide the exact solution in transient cases with independence of the grid and also to converge to the exact solution in transient problems at a high rate as the grid is refined.

For transient problems in the framework of the SWE, different approaches can be found in the literature regarding the treatment of the source term contact discontinuity. Two main tendencies are observed in the literature: one is based on energy and mass conservation and the other one based on mass and momentum conservation. For instance, some authors [29,30] claim that energy must always be conserved since the bed step discontinuity is a contact wave and Riemann