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Sensor placement for calibration of spatially varying model parameters

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A B S T R A C T

This paper presents a sensor placement optimization framework for the calibration of spatially varying model parameters. To account for the randomness of the calibration parameters over space and across specimens, the spatially varying parameter is represented as a random field. Based on this representation, Bayesian calibration of spatially varying parameter is investigated. To reduce the required computational effort during Bayesian calibration, the original computer simulation model is substituted with Kriging surrogate models based on the singular value decomposition (SVD) of the model response and the Karhunen–Loeve expansion (KLE) of the spatially varying parameters. A sensor placement optimization problem is then formulated based on the Bayesian calibration to maximize the expected information gain measured by the expected Kullback–Leibler (K–L) divergence. The optimization problem needs to evaluate the expected K–L divergence repeatedly which requires repeated calibration of the spatially varying parameter, and this significantly increases the computational effort of solving the optimization problem. To overcome this challenge, an approximation for the posterior distribution is employed within the optimization problem to facilitate the identification of the optimal sensor locations using the simulated annealing algorithm. A heat transfer problem with spatially varying thermal conductivity is used to demonstrate the effectiveness of the proposed method.

1. Introduction

Model calibration is the process of estimating the values of model parameters based on observations. Various approaches for model calibration have been studied in the past such as least squares regression [1], maximum likelihood estimation (MLE) [2], and Bayesian method [3]. Both the least squares and MLE methods return point estimates of the calibration parameters, whereas the Bayesian approach gives posterior distributions of the calibration parameters. Bayesian calibration was extended by Kennedy and O'Hagan (KOH) to incorporate various sources of uncertainty [4]. This framework has been applied to various engineering problems [3,5,6] and has been followed and further investigated by many other researchers [7–11].

All the above studies have considered spatially constant model parameters in calibration. For some systems, this idealizes heterogeneous systems as a homogenized medium. Considering model parameters as spatially varying (as opposed to spatially constant) enables us to perform model prediction more accurately due to the lesser restrictive assumption [12,13].

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In addition, the spatially varying parameter in reality varies both in space and between specimens. Model parameters that vary spatially within a specimen and also vary across specimens are referred to as *spatially varying random parameters* in the rest of this paper.

Two issues related to spatially varying parameters are addressed in this paper: (1) Bayesian calibration of spatially varying random parameters, and (2) the optimization of sensor placement. Due to the involvement of the spatially varying random parameters, several challenges are introduced in both surrogate modeling and likelihood computation during Bayesian calibration. In addition to the challenges in Bayesian calibration, a large number of available sensor locations makes the selection of optimal locations computationally challenging for this discrete optimization problem. Details of the challenges related to these two issues are explained below.

Bayesian calibration usually requires thousands of function evaluations; therefore inexpensive surrogate models are often used to replace the computationally expensive physics simulation models. For Bayesian calibration with spatially constant parameters, the surrogate model can be built directly. However, constructing the surrogates with spatially varying random parameters is challenging due to the variation of model response over time, space, and specimens. Building a single surrogate model for such a case can be inaccurate as the optimal characteristics for one output may not be the same for the others [14]. Building correlated surrogate models for the vector of responses by using techniques such as co-kriging [15] can handle only a small number of outputs at present. If many independent surrogate models are built without considering correlation between the responses, the surrogate models cannot represent the original model accurately. In addition to the above difficulty, calibration of the spatially varying random parameters is also more challenging than the calibration of spatially constant parameters. The uncertainty in the surrogate model, the model discrepancy of the simulation model, and the variations as well as correlations over time, space, and specimens need to be considered during model calibration.

The other critical issue that needs to be addressed is where the sensors should be placed in order to effectively collect data for model calibration. Though sensor location optimization has been studied in the literature, such studies have mostly focused on structural health monitoring (SHM) and structural control system design [16–18], and are mostly deterministic. In the context of experiment design for model calibration, most studies have focused on systems with spatially constant model parameters [19–23]. For systems with spatially constant model parameters, optimizing the location of a single sensor may be sufficient as shown by Huan and Marzouk [21]. However, in the case of spatially varying random parameters, a single sensor is not enough since it is impossible for one sensor to capture the correlation information of the parameter over space. Multiple sensors (spread over the spatial domain) are required. This makes sensor placement optimization more challenging. When model calibration is embedded within the sensor placement optimization framework, this makes the optimization more complicated than that of spatially constant parameters. For this discrete optimization problem, a large number of solutions (i.e., spatial locations and time instants where observation data can be collected) need to be explored which makes it computationally challenging.

The focus of this paper is to develop a methodology for the *calibration of spatially varying random parameters* and the *optimum selection of sensor locations* among a large number of candidate locations. Both variations over space and across specimens are considered during the sensor placement optimization. The challenges discussed above can be summarized as: (1) How to effectively build a surrogate model for the high-dimensional response with spatially varying random parameters? (2) How to perform Bayesian calibration for problems with spatially varying random parameters and account for correlations and various sources of uncertainty during the calibration? (3) How to efficiently solve the sensor placement optimization problem?

In this paper, the spatially varying random parameter is modeled as a random field. To address the first challenge (i.e. surrogate modeling), the high-dimensional output is mapped to a low-dimensional latent space through singular value decomposition (SVD) [24,25] and then surrogate model is built in the latent space. A similar idea has been implemented by Jia et al. [14] for the risk assessment of hurricane events, and by Hombal and Mahadevan [26] for crack growth prediction, both of which are forward uncertainty propagation problems. We extend that idea here to the inverse problem of calibration. To handle the second challenge, the likelihood function is evaluated by integrating out the random input variables introduced by the variability over specimens. After addressing the first two challenges, an optimization problem is then formulated based on the surrogate modeling and Bayesian calibration method, to select the optimal sensor placement by maximizing the expected information gain. A Gaussian copula [27] function is used for fast estimation of the objective function. The optimization problem (i.e. the third challenge) is then solved using a simulated annealing algorithm.

The contributions of this paper are summarized as: (1) development of a framework for the calibration of spatially varying random parameters; (2) construction of surrogate models for response with spatially varying random model parameters and using them for sensor placement optimization; (3) formulation of an optimization problem for selecting optimal sensor placement locations; and (4) a solution procedure for solving the sensor placement optimization problem.

The remainder of this paper is organized as follows. Section 2 briefly introduces background on Bayesian calibration and discusses the challenges in the calibration of spatially varying random model parameters. Section 3 develops the proposed sensor location optimization methodology. Section 4 provides an illustrative example, and Section 5 gives conclusions.

2. Background

This section discusses Bayesian calibration and establishes the motivation for the proposed methods.