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A reformulation of the conservative level set reinitialization equation for accurate and robust simulation of complex multiphase flows

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ABSTRACT

This paper presents an accurate and robust reinitialization equation for the conservative level set that does not significantly deform stationary surfaces. The compression and diffusion term of the reinitialization equation are reformulated to use a distance level set directly mapped from the conservative level set. The normals are calculated using a distance level set reconstructed from the interface using a fast marching method, increasing robustness and allowing the use of high order, non-TVD transport schemes. Using this new reinitialization equation, we present results for canonical test cases, such as Zalesak's disk and spurious currents, which show significant improvement. A simulation of a liquid–gas jet with $Re = 5000$ is also presented to demonstrate the volume conservation properties of the method in more complex flows.

1. Introduction

Multiphase flows are ubiquitous in nature and exist in many engineering applications, making their simulation very important. However, there are also many numerical difficulties associated with simulating multiphase flows that are not present in single fluid flows. With the presence of multiple phases comes the requirement to track the interface between the phases. Across this interface exists discontinuities in the physical properties of the phases and a singular force due to surface tension, both of which require special consideration when solving the equations in a discrete manner.

Level set methods [1] are often used to track this interface by representing it with a scalar field. The original level set method used a signed distance field, $\phi(\mathbf{x}, t)$, defined as the distance from $\mathbf{x}$ to the nearest point on the interface Γ at time t. This scalar field is advected with the flow velocity, $\mathbf{u}$, to effectively transport the interface with

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = 0\,. \tag{1}$$

Through choosing a sign for the distance based on the local phase, the interface becomes implicitly represented as the zero iso-contour of ϕ. The transport will lead to errors in ϕ, requiring a reinitialization equation to reshape ϕ into a proper distance function, for which $|\nabla \phi| = 1$. Neither the transport, Eq. (1), nor the reinitialization equation, are conservative, which leads to significant volume conservation errors, requiring additional attention and techniques to improve volume conservation. This is a key issue with the original level set methods, preventing their use for a wide class of problems, such

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as turbulent atomization. For the remainder of this paper, these original level set methods, which transport and reinitialize the signed distance field $\phi(\mathbf{x},t)$, will be referred to as distance level set (DLS) methods.

In order to better conserve the enclosed volume, Olsson and Kreiss [2] and Olsson et al. [3] proposed using a conservative level set (CLS) function, instead of ϕ, to implicitly track the interface. This function is defined as

$$\psi(\mathbf{x},t) = \frac{1}{2}\left(\tanh\left(\frac{\phi(\mathbf{x},t)}{2\epsilon}\right)+1\right), \tag{2}$$

where ψ is the CLS and ϵ is a free parameter that determines the interface thickness. With this mapping from ϕ, the interface is now represented by the 0.5 iso-contour of ψ.

The conservative level set function, Eq. (2), can be seen as a smooth approximation of the sharp Heaviside function, $H(\phi)$, which has a value of 1 in one phase ($\phi > 0$), and 0 in the other ($\phi < 0$). As is clear, the sharper the hyperbolic tangent profile, i.e., the smaller the value of ϵ, the better the approximation of $H(\phi)$. The exact enclosed volume is given by the integration of the Heaviside function,

$$\mathcal{V}_l = \int H(\phi)d\mathcal{V} = \int H\left(\psi - \frac{1}{2}\right)d\mathcal{V}. \tag{3}$$

Similarly, the approximation of the enclosed volume using the conservative level set function is given by

$$\tilde{\mathcal{V}}_l = \int \psi d\mathcal{V}, \tag{4}$$

with $\lim_{\epsilon\to 0} \tilde{\mathcal{V}}_l = \mathcal{V}_l$. A conservative discretization of the ψ equations (transport and reinitialization) leads to exact conservation of $\tilde{\mathcal{V}}_l$, but only approximate conservation of $\mathcal{V}_l$. The capability of the method to conserve enclosed volume, i.e. the accuracy with which $\tilde{\mathcal{V}}_l$ approximates $\mathcal{V}_l$, is a central focus of this paper.

Assuming incompressibility, ψ is transported, using a conservative finite volume discretization, as

$$\frac{\partial \psi}{\partial t} + \nabla\cdot(\mathbf{u}\psi) = 0, \tag{5}$$

and, after transporting, reinitialized back to its proper hyperbolic tangent profile, with a conservative finite volume discretization, through

$$\frac{\partial \psi}{\partial \tau} = \nabla\cdot\left(\epsilon\left(\nabla\psi\cdot\mathbf{n}\right)\mathbf{n} - \psi\left(1-\psi\right)\mathbf{n}\right), \tag{6}$$

where $\mathbf{n}$ is the interface normal vector calculated using

$$\mathbf{n} = \frac{\nabla\psi}{|\nabla\psi|}. \tag{7}$$

With ϵ controlling the interface profile thickness, and a smaller value of ϵ resulting in better volume conservation properties, it is desirable to set ϵ to a small value, typically $\epsilon = \Delta x/2$ [2]. This causes sharp gradients, which can produce overshoots and undershoots of ψ, causing $\mathbf{n}$ to switch directions when calculated with Eq. (7). Olsson and Kreiss [2] handled this problem through the use of low order, total variation diminishing (TVD) schemes for transport to prevent the creation of new local extrema, however, this significantly reduces the overall accuracy of the method. This method will be referred to as the conservative level set (CLS) method.

In order to alleviate the need to use TVD transport schemes, the accurate conservative level set (ACLS) method [4] uses a fast marching method (FMM) [5] to reconstruct a signed distance function, ϕ_{FMM}, from the 0.5 iso-contour of ψ, and calculates the normals using ϕ_{FMM} as

$$\mathbf{n} = \frac{\nabla\phi_{\text{FMM}}}{|\nabla\phi_{\text{FMM}}|}. \tag{8}$$

This prevents oscillatory transport errors in ψ from spuriously impacting the orientation of normals, removing the prior need to use TVD transport schemes. Because of this, inexpensive and high order accurate schemes can be used. At the same resolution, the ACLS method can therefore track finer interfacial structures than the original CLS method. Additionally, it was found that the use of high order transport schemes significantly improves volume conservation [4].

The reinitialization of ψ using Eq. (6), which is used in both the CLS method [2,3] and the ACLS method [4], can still be a significant source of numerical errors. As an example, Fig. 1 shows a spherical drop resolved by $D/\Delta x = 20$, reinitialized in place with Eq. (6), updating the normals after each iteration. Without numerical dissipation from transport, errors from the reinitialization equation accumulate and lead to significant deformation of the interface, as demonstrated in Fig. 1(b), showing the deformation of the sphere after 1000 iterations. The obvious solution would be to reduce or eliminate