Journal of Computational Physics 343 (2017) 217–234

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Analysis and numerical solution of transient electromagnetic scattering from two cavities

Richard Uber [a,*], Aihua Wood [a], Michael Havrilla [b]

[a] Department of Mathematics and Statistics, Air Force Institute of Technology, 2950 Hobson Way, Wright-Patterson AFB, OH 45433, United States

[b] Department of Electrical and Computer Engineering, Air Force Institute of Technology, 2950 Hobson Way, Wright-Patterson AFB, OH 45433, United States

ARTICLE INFO

Article history:
Received 26 May 2016
Received in revised form 23 December 2016
Accepted 15 April 2017
Available online 20 April 2017

Keywords:
Electromagnetic scattering
Half-plane Green's function
Cavities
Finite-element boundary-integral method
Newmark method
Transient scattering

ABSTRACT

This paper presents a finite-element boundary integral method to model transient scattering from two cavities embedded in a ground plane. Multiple-cavity scattering in the time domain both increases the computational complexity of the scattering problem and introduces cross-talk between scatterers. By modeling the cavities together, coupling effects are enforced directly and multiple reflections are readily observable. The variational formulation is shown to be well-posed for each time step and a stable numerical implementation for rectangular cavities is demonstrated.

Published by Elsevier Inc.

1. Introduction

Electromagnetic scattering from open cavities and cavity-backed apertures has been an active area of research with applications in aircraft design and radar cross-section (RCS) modeling. Major scattering sources can be separated into the following categories: reentrant structures, specular scattering, traveling wave echoes, edge and vertex diffraction, creeping waves, interactions, and surface discontinuities [1], of which the greatest contributors are reentrant structures and specular scattering. As RCS reduction methods are growing in sophistication, understanding lesser scattering sources, such as edge diffraction, interactions, and discontinuities, becomes of greater importance. A few examples of necessary aircraft features that create gaps or cavities are control surfaces, access panels, ordnance bay doors, engine inlets [2], exhaust nozzles [3], etc. Examining influence of cavity spacing and relevant interactions may provide insight about how to maximize or minimize constructive or destructive effects on the scattered fields. Additionally, the electromagnetic environment has become more complex, and engineers must now consider a wider range of frequencies than in the past. A structure which may respond like a gap or crack at one frequency range [4], may function like a reentrant structure at a much higher frequency. Transient analysis of scattering is useful in this case because it generates a wide-band solution and may capture potential non-linear behavior and interactions not visible in time-harmonic solutions.

Recent work on this topic has focused on the finite-element boundary integral (FE-BI) method [5] as an efficient way to couple the interior cavity region, solved numerically with the finite element method, with the exterior region which

* Corresponding author.
E-mail addresses: richard.uber@afit.edu (R. Uber), aihua.wood@afit.edu (A. Wood), michael.havrilla@afit.edu (M. Havrilla).

http://dx.doi.org/10.1016/j.jcp.2017.04.043
0021-9991/Published by Elsevier Inc.

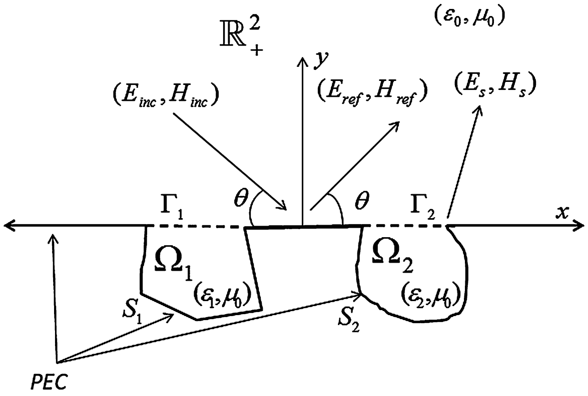

Fig. 1. Two cavity environment.

is solved analytically with a Green's function. Transient scattering from a single cavity in three dimensions was analyzed in [6], and numerical modeling of the two-dimensional problem was described in [7], utilizing the Newmark method for time-stepping. The Newmark method discretizes the time-variant equations into forced time-harmonic Helmholtz equations which are solved for each time step. Numerical handling of the multiple-cavity time-harmonic problem was well explained in [8], along with proof of well-posedness of the problem. The finite element method is utilized due to its flexibility in conforming to different geometries as well as the ease of imposing the boundary conditions.

The time-harmonic scattering of a cavity embedded in the ground plane is well described in [8] and [5]. For high frequencies or large cavities, [9] presents a highly accurate computational method, while bounds and stability are addressed in [10,11] and [12]. For overfilled cavities, such that the material filling protrudes somewhat from the cavity opening, or cavities with non-flush edge conditions, the FE-BI method is still appropriate as in [13–15]. The difference being that when the cavity is flush with the ground plane, the transparent boundary can be drawn at the aperture itself and using the half-space Green's function for the exterior solution, whereas with an overfilled cavity, a semi-circular boundary must be drawn and the known Green's function for the semi-circle used for the exterior solution. A straightforward proof of existence and uniqueness of a solution to the single-cavity time-harmonic problem is presented in [16] utilizing the Fredholm Alternative Theorem. Additionally, transient scattering from a single overfilled cavity in an impedance ground plane is examined in [17].

In [7], a thorough analysis of the one-cavity problem in the time domain is conducted with the key result being that the variational formulations of the TM^z and TE^z problems have unique weak solutions for each discretized time step. In general, the multiple cavity problem has many parallels with the single-cavity problem. The primary difference that is faced in the multiple cavity problem is the coupling between cavities. Time-harmonic analysis of multiple-cavity scattering is conducted in [8,18] and an analytical solution for multiple rectangular cavities is presented in [19] using Fourier transform methods.

This work extends the findings in [7,8] to the transient, two-cavity problem. The key result is that the variational formulation for transient scattering from two cavities embedded in a ground plane using the Newmark Method is well-posed. Numerical examples are provided to demonstrate the application of the proposed method.

2. Problem statement

2.1. Problem environment

This work considers an infinite perfectly-conducting ground plane (PEC) with two fully-embedded two-dimensional cavities. In three dimensions, these cavities are z-invariant infinite grooves, or channels. Orientation of the fields will be TM^z, indicating that wave propagation and the magnetic field will be fully described in the x–y plane, and the electric field will be fully z-directed, but described as a function of x and y. The area above the ground plane will be considered free space $\mathbb{R}^2_+ = \{x, y \in \mathbb{R}^2 : y \geq 0\}$ with homogeneous parameters ε_0, μ_0 describing the permittivity and permeability of free space. The cavities themselves will be denoted by Ω_1, Ω_2. Within each cavity, there may be a non-magnetic, homogeneous, dielectric filler, with parameters μ_0, ε_j with $\varepsilon_j \in \mathbb{C}, \mathrm{Re}\{\varepsilon_j\} > 0$, $\mathrm{Im}\{\varepsilon_j\} = 0$. PEC cavity walls are denoted S_1, S_2 and cavity apertures denoted Γ_1, Γ_2, respectively for cavities 1 and 2. This environment is illustrated in Fig. 1.

2.2. Transient scattering problem

The problem is posed as an initial-boundary value problem. Given an incident TM^z electric field $u^{inc}(x, y, t)$ defined in $\mathbb{R}^2_+ \times t \geq 0$, find the total field $u(x, y, T)$ for a given time $T > 0$ and $(x, y) \in \mathbb{R}^2_+$ subject to: