Journal of Computational Physics 345 (2017) 1–16

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Robust integral formulations for electromagnetic scattering from three-dimensional cavities

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ARTICLE INFO

Article history:
Received 11 June 2016
Received in revised form 5 May 2017
Accepted 6 May 2017
Available online 11 May 2017

Keywords:
Electromagnetic
Cavity scattering
Maxwell
Integral equation
Axisymmetric

ABSTRACT

Scattering from large, open cavity structures is of importance in a variety of electromagnetic applications. In this paper, we propose a new well conditioned integral equation for scattering from general open cavities embedded in an infinite, perfectly conducting half-space. The integral representation permits the stable evaluation of both the electric and magnetic field, even in the low-frequency regime, using the continuity equation in a post-processing step. We establish existence and uniqueness results, and demonstrate the performance of the scheme in the cavity-of-revolution case. High-order accuracy is obtained using a Nyström discretization with generalized Gaussian quadratures.

1. Introduction

The computation of electromagnetic wave propagation in the presence of large, open cavities is an important modeling task. It is critical, for example, in understanding the effect of exhaust nozzles and engine inlets on aircraft, as well as surface deformations in automobiles and other land-based vehicles [4,8,29,31,33]. The presence of such structures plays a dominant role in both the near field, where electromagnetic interference is of concern, and in the far field, where the radar cross-section can be used for identification and classification (including stealth-related calculations). Fast and accurate solvers to simulate such scattering phenomena are essential for both design optimization and verification.

A variety of numerical methods have been proposed to solve such scattering problems. Largely speaking, they fall into two categories. The first is direct numerical simulation using finite difference [8], finite element [33], mode-matching [5] and boundary integral methods [8,38]. The second is asymptotic methods, including Gaussian beam approximations [13] and physical optics-based schemes [32]. The latter methods tend to work well at very high frequencies in the absence of multiple near-field scattering events, and are generally not well suited for high-precision calculations in geometrically complex environments. Solving the governing Maxwell equations using finite difference and finite element methods, on the other hand, requires the discretization of an unbounded domain. In practice, these methods must either employ approximate outgoing boundary conditions to mimic the radiation condition at infinity, or be coupled with a boundary integral representation beyond some distance so that the radiation condition is satisfied exactly. In the present work, we will focus on boundary integral equation methods since they are free from grid-based numerical dispersion, can achieve high-order accuracy in complex geometry, and require degrees of freedom only on the boundary of the scatterer itself, greatly reducing

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http://dx.doi.org/10.1016/j.jcp.2017.05.008
0021-9991/

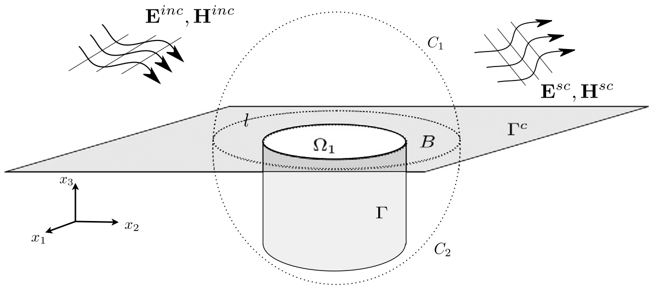

Fig. 1. A cavity Ω_1 in a perfectly conducting half-space $x_3 \geq 0$, with boundary Γ. The surfaces C_1 and C_2 define the two halves of a sphere C which is sufficiently large to contain the cavity and a finite buffer region, denoted by B, in the x_1x_2-plane beyond the edge of the cavity. The curve ℓ denotes the outer edge of the buffer region B. The unbounded half-space boundary outside of C is denoted by Γ^c.

the number of unknowns. The Green's function used to represent the solution satisfies the outgoing (radiation) condition exactly. Existing integral representations for cavity problems, however, generally yield integral equations of the first-kind [1]. First-kind equations can lead to ill-conditioned discrete linear systems, especially if substantial mesh refinement is required. Refined meshes may be needed, for example, to resolve geometric singularities. Furthermore, several existing formulations also suffer from *spurious resonances*, including those of mixed first-/second-kind systems [38]. Finally, because of the nature of the dyadic Green's function for the electric field, standard methods based on discretizing the physical electric current also suffer from low-frequency breakdown [16,40]. This behavior is discussed in more detail below.

In this paper, we propose a new representation of the scattered field that leads to a well-posed (resonance-free) integral equation which is immune from low-frequency breakdown. This allows for a stable numerical discretization along arbitrarily adaptive meshes. In our numerical examples, using the fact that the scatterer (i.e. the cavity) is axisymmetric permits us to use separation of variables in cylindrical coordinates, applying the Fourier transform in the angular (azimuthal) variable. This procedure leads to a sequence of uncoupled two-dimensional boundary integral equations on the generating curve that defines the cross-section of the boundary of the scatterer (see Fig. 3). There are various numerical technicalities associated with implementing body-of-revolution integral equation solvers, and we do not seek to review the substantial literature here. We instead refer the reader to [23,24,28,34,39] and the references therein. A concise overview of the discretization and resulting solver is given in Section 5. Similar high-order techniques have been applied to solve the Helmholtz equation on surfaces of revolution [23,24,34,39] and the full Maxwell equations (for closed-cavity resonance problems) in [25].

Due to the applicability of cavity scattering in physics and engineering, there has been much work dedicated to both the mathematical and numerical aspects of the problem. The well-posedness of the (forward) scattering problem is discussed in [1,2] in the case of the two-dimensional problem, and in [3] for the three-dimensional case. The paper [9] provides the explicit dependence of the scattered field on the wavenumber in the high-frequency context. In [4,30], the authors studied uniqueness and stability issues for the inverse problem, where one seeks to recover the shape of an unknown cavity using near-field data. The corresponding optimal design problem, i.e. to find a cavity shape that minimizes the radar cross section, under certain constraints, is discussed in [6,7] in the two-dimensional setting.

An outline of the paper follows: Section 2 provides a detailed introduction to the problem of scattering from an open cavity and proposes an integral representation that leads to a well conditioned integral formulation. In Section 3, we prove that this integral equation has a unique solution for a given incident field. In Section 4, we show how to avoid low-frequency breakdown merely by the use of various vector identities and physical considerations. In Section 5, we briefly discuss the separation of variables solver for axisymmetric cavities, and then illustrate its accuracy and stability in Section 6. Section 7 contains a brief discussion of open problems and some concluding remarks.

2. Mathematical formulation of the scattering problem

Suppose now that a perfectly conducting cavity Ω_1 extends into the lower half-space $x_3 < 0$, as depicted in Fig. 1. See the caption of Fig. 1 for a description of the geometrical setup. The region in the lower half-space with boundary $\Gamma \cup B \cup \Gamma^c$ is assumed to be perfectly conducting. Given a time harmonic incident field $(\tilde{\mathbf{E}}^{\text{inc}}, \tilde{\mathbf{H}}^{\text{inc}})$ with an assumed time dependence of $e^{-i\omega t}$, we seek to find the scattered field $(\tilde{\mathbf{E}}^{\text{sc}}, \tilde{\mathbf{H}}^{\text{sc}})$ so that the total field

$$\tilde{\mathbf{E}} = \tilde{\mathbf{E}}^{\text{inc}} + \tilde{\mathbf{E}}^{\text{sc}}, \quad \tilde{\mathbf{H}} = \tilde{\mathbf{H}}^{\text{inc}} + \tilde{\mathbf{H}}^{\text{sc}}$$

satisfies the Maxwell equations

$$\nabla \times \tilde{\mathbf{E}} - i\omega\mu\, \tilde{\mathbf{H}} = 0,$$
$$\nabla \times \tilde{\mathbf{H}} + i\omega\varepsilon\, \tilde{\mathbf{E}} = 0$$

for $\mathbf{x} \in \mathbb{R}^{3+} \cup \Omega_1$. The material parameters are given by ε, the electric permittivity, and μ, the magnetic permeability. Assuming ε and μ are constant, it is convenient to denote suitably normalized fields by $\mathbf{E} = \sqrt{\varepsilon}\tilde{\mathbf{E}}$, $\mathbf{H} = \sqrt{\mu}\tilde{\mathbf{H}}$, etc., leading to a simpler form of Maxwell's equations