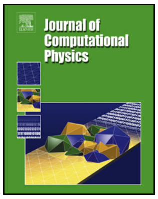
Journal of Computational Physics 345 (2017) 17–32

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp



Numerical solution of a coefficient inverse problem with multi-frequency experimental raw data by a globally convergent algorithm

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ARTICLE INFO

Article history:
Received 30 November 2016
Received in revised form 5 May 2017
Accepted 8 May 2017
Available online 11 May 2017

Keywords:
Experimental multi-frequency data
Backscatter data
Raw data
Coefficient inverse problem
Globally convergent numerical methods

ABSTRACT

We analyze in this paper the performance of a newly developed globally convergent numerical method for a coefficient inverse problem for the case of multi-frequency experimental backscatter data associated to a single incident wave. These data were collected using a microwave scattering facility at the University of North Carolina at Charlotte. The challenges for the inverse problem under the consideration are not only from its high nonlinearity and severe ill-posedness but also from the facts that the amount of the measured data is minimal and that these raw data are contaminated by a significant amount of noise, due to a non-ideal experimental setup. This setup is motivated by our target application in detecting and identifying explosives. We show in this paper how the raw data can be preprocessed and successfully inverted using our inversion method. More precisely, we are able to reconstruct the dielectric constants and the locations of the scattering objects with a good accuracy, without using any advanced *a priori* knowledge of their physical and geometrical properties.

1. Introduction

We are interested in a Coefficient Inverse Problem (CIP) with real world applications, including the detection and identification of explosives, nondestructive testing and material characterization. A new globally convergent numerical method for solving such a CIP has been recently developed by our group in [1]. Numerical study in [1] was conducted for computationally simulated data. In this paper, we study the performance of this method for the case of experimental raw data. These data were collected using a microwave scattering facility at the University of North Carolina at Charlotte.

More precisely, we consider the CIP of reconstruction of physical and geometrical properties of three-dimensional objects from experimental multi-frequency data without using any detailed *a priori* knowledge of those objects. Our study is mainly motivated by potential applications in detection and identification of explosives such as, e.g., anti-personnel mines and improvised explosive devices (IEDs), see [2,3]. These targets are placed in air, and the measured data are the backscatter

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http://dx.doi.org/10.1016/j.jcp.2017.05.015

corresponding to a single incident wave at multi-frequencies. In addition, we note that IEDs are also often buried under the ground for which one needs to study the corresponding inverse problem of determining buried objects. This topic is subject to a future publication.

The idea here is to determine the dielectric constants of targets. The knowledge of dielectric constants might serve in the future classification algorithms as a piece of information, which would be an additional one to those commonly currently used in the radar community. Indeed, this community relies now only on the intensity of radar images, see, e.g., [4,5]. It is well known that the question of a reliable differentiation between explosives and clutter is not yet addressed satisfactory in the radar community. So, estimates of dielectric constants and shapes of targets combined with the image intensity and other parameters might lead in the future to such classification algorithms, which would address this question better. The second potential application of our study is in nondestructive testing and materials characterization.

A Coefficient Inverse Problem is the problem of recovering a coefficient of a partial differential equation from boundary measurements of its solutions. There is a large body of the literature on imaging methods for reconstructing geometric information about targets, such as their shapes, sizes and locations. We refer to, e.g., [6–15] and references therein for well-known imaging techniques in inverse scattering such as level set methods, sampling methods, expansion methods, and shape optimization methods. In particular, we also refer to [16], where imaging algorithms are designed and their performance with respect to noise are analyzed in the frameworks of small and extended scatterers. However, for detecting or identifying, for instance, IEDs, the physical properties of the targets (in our model, the dielectric constants), would play a more important role [3]. Furthermore, determining the spatially distributed dielectric constants, which is of our main interest, is known to be a more difficult task since CIPs are, in general, highly nonlinear and severely ill-posed.

Among the inversion methods developed for solving the CIPs, the two probably most well known approaches are nonlinear approximation schemes and weak scattering approximation methods such as Born approximation and physical optics. The weak scattering approximation is not applicable to the inverse problem under consideration, where the scattering objects can be strong scatterers. Regarding the nonlinear optimization approaches or also known as iterative solution methods, we refer to, e.g., [17–19] and references therein. It is well-known that the convergence for this class of methods typically requires a good *a priori* initial approximation of the exact solution, that is, the starting point of iterations should be chosen to be sufficiently close to the solution. Hence, we call such methods *locally convergent*. We note that in our desired applications such *a priori* knowledge is not always available.

This limitation of nonlinear optimization approaches is avoided in the so-called approximately globally convergent method (globally convergent method, for short, or GCM), which has been recently introduced, see [20]. The GCM, which does not use optimization schemes, aims to provide a good approximation to the solution of the coefficient inverse problem without using any advanced *a priori* knowledge of the solution. More precisely, the concept “approximate globally convergence” can be understood in the language of functional analysis as follows: under a reasonable approximate mathematical assumption the method provides at least one point in a sufficiently small neighborhood of the exact coefficient without *a priori* knowledge of any point in this neighborhood. The accuracy of the approximation or the distance between those points and the exact solution depends on the error in the data and some parameters of the discretization. We point out that the fact of the proximity of that point to the correct coefficient, which was achieved without any *a priori* knowledge of that small neighborhood, is the main advantage of our globally convergent method over locally convergent ones. Indeed, as soon as one knows a point in a sufficiently small neighborhood of the true solution, one can refine it via a locally convergent method, see, e.g., [20, Chapter 4]. The latter, however, is outside of the scope of the current publication. We refer to [20, Theorem 2.9.4] for more details about the definition of the global convergence as well as a rigorous mathematical analysis of the global convergence of the method relying on an approximate mathematical framework.

In previous works of the GCM summarized in [20], the model of hyperbolic wave type equations is considered and the time-domain problem is converted into the pseudo-frequency domain problem via the Laplace transform. Since the Laplace transform used in the method has an exponentially decaying kernel, one likely loses some information in taking this transform of the (far field) measured data. Therefore, we exploit the Fourier transform to improve the performance of the GCM and to extend its direct application to multi-frequency data, which is common in applications to materials characterization. This leads us to study a *new* GCM in [1]. More precisely, in the latter paper, we developed a GCM for solving the CIP for the Helmholtz equation with multi-frequency data. The main difficulty in developing this new GCM is to work with complex-valued functions where the maximum principle, which plays an important role in the previous GCM [20], is no longer applicable.

As a continuation of the work of [1], the goal of this paper is to analyze the performance of our new GCM for multi-frequency experimental raw data. There are some major new features of the present paper:

i. The globally convergent approach together with its advantages makes this work different from previously known locally convergent methods.
ii. This paper is the first one where we study the experimental multi-frequency data for the new GCM of [1].
iii. For the multi-frequency raw data in this paper, we developed a new data preprocessing procedure, which is discussed later in this section. This procedure is substantially different from that of the time domain data in [21].
iv. Recall that this is the CIP with a minimal amount of multi-frequency raw data (backscatter data associated to a single incident wave) and we are not aware of any literature that addresses the numerical solution of this problem without using any advanced *a priori* knowledge of the solution.