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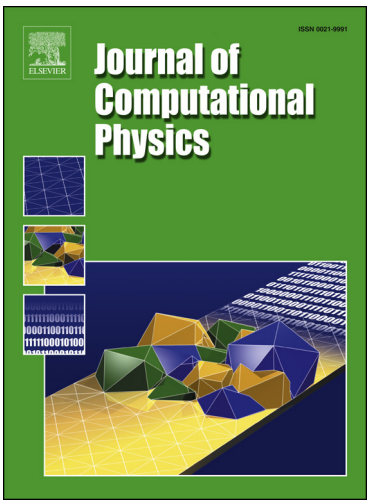

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High-order Finite-volume Solutions of the Steady-state Advection-diffusion Equation with Nonlinear Robin Boundary Conditions

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Abstract

We propose high-order finite-volume schemes for numerically solving the steady-state advection-diffusion equation with nonlinear Robin boundary conditions. Although the original motivation comes from a mathematical model of blood clotting, the nonlinear boundary conditions may also apply to other scientific problems. The main contribution of this work is a generic algorithm for generating third-order, fourth-order, and even higher-order explicit ghost-filling formulas to enforce nonlinear Robin boundary conditions in multiple dimensions. Under the framework of finite volume methods, this appears to be the first algorithm of its kind. Numerical experiments on boundary value problems show that the proposed fourth-order formula can be much more accurate and efficient than a simple second-order formula. Furthermore, the proposed ghost-filling formulas may also be useful for solving other partial differential equations.

Keywords: nonlinear Robin boundary conditions, the steady-state advection-diffusion equation, multivariate interpolation, poised stencils, QR factorization, blood clotting.

1. Introduction

Consider the steady-state advection-diffusion equation

$$-\nabla \cdot (\mathbf{u}\phi) + \nabla \cdot (\nu \nabla \phi) = 0 \tag{1}$$

on a rectangular domain $\Omega \subset \mathbb{R}^D$ with constant diffusivity ν and a nonlinear Robin boundary condition of the form

$$\alpha\phi + \beta\frac{\partial \phi}{\partial n} + \lambda\phi\frac{\partial \phi}{\partial n} = \chi \qquad \text{on } \partial\Omega, \tag{2}$$

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