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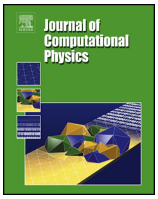

An implicit turbulence model for low-Mach Roe scheme using truncated Navier–Stokes equations

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ABSTRACT

The original Roe scheme is well-known to be unsuitable in simulations of turbulence because the dissipation that develops is unsatisfactory. Simulations of turbulent channel flow for $\mathrm{Re}_\tau = 180$ show that, with the 'low-Mach-fix for Roe' (LMRoe) proposed by Rieper [J. Comput. Phys. 230 (2011) 5263–5287], the Roe dissipation term potentially equates the simulation to an implicit large eddy simulation (ILES) at low Mach number. Thus inspired, a new implicit turbulence model for low Mach numbers is proposed that controls the Roe dissipation term appropriately. Referred to as the automatic dissipation adjustment (ADA) model, the method of solution follows procedures developed previously for the truncated Navier–Stokes (TNS) equations and, without tuning of parameters, uses the energy ratio as a criterion to automatically adjust the upwind dissipation. Turbulent channel flow at two different Reynold numbers and the Taylor–Green vortex were performed to validate the ADA model. In simulations of turbulent channel flow for $\mathrm{Re}_\tau = 180$ at Mach number of 0.05 using the ADA model, the mean velocity and turbulence intensities are in excellent agreement with DNS results. With $\mathrm{Re}_\tau = 950$ at Mach number of 0.1, the result is also consistent with DNS results, indicating that the ADA model is also reliable at higher Reynolds numbers. In simulations of the Taylor–Green vortex at $\mathrm{Re} = 3000$, the kinetic energy is consistent with the power law of decaying turbulence with -1.2 exponents for both LMRoe with and without the ADA model. However, with the ADA model, the dissipation rate can be significantly improved near the dissipation peak region and the peak duration can be also more accurately captured. With a firm basis in TNS theory, applicability at higher Reynolds number, and ease in implementation as no extra terms are needed, the ADA model offers to become a promising tool for turbulence modeling.

1. Introduction

Recently, much attention has been centered on compressible turbulent flows at low Mach numbers, such as encountered in aeroacoustics, combustion, and significant heat transfer. Concerning simulations in these areas, the direct use of compressible solvers, e.g., the Roe scheme [1], is unsuitable because energy dissipation performed within the numerical scheme was originally designed for shock waves. Several related numerical methods have been proposed to solve this issue.

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The first study by Turkel [2] generalizes the artificial compressibility method originally proposed by Chorin [3]. Applying a preconditioning matrix to both incompressible and compressible systems accelerated convergence. This method extends the applicable range of compressible flow equations to low Mach numbers. Weiss and Smith [4] applied the Roe scheme with a preconditioning matrix to develop solutions of the three-dimensional Navier–Stokes equations for low Mach number flows. Thornber [5] uses the local Mach number to scale appropriately the velocity jump at the cell interface to minimize the excessive numerical dissipation on reconstruction. Despite the slight additional computational expense, the noted advantage of this modification is the absence of a cut-off Mach number, which is generally needed for the preconditioning method. Instead of modifying the reconstruction such as [5], Rieper [6] adopted the local Mach number to rescale directly the velocity jump in the characteristic values in the Roe scheme. This simple fix, called 'low-Mach-fix for Roe' (LMRoe), can be applied to all reconstructions such as the monotonic upstream-centered scheme for conservation laws (MUSCL), essentially non-oscillatory (ENO) and weighted ENO (WENO) or discontinuous Galerkin methods. For those adopting the Roe scheme, this fix appeared more general than Thornber's approach [5]. Because of its advantages of generality, easy implementation, and also absence of a cut-off Mach number, LMRoe has been applied to diverse fluid problems such as wind turbine flows [7]. However, we believe a more detailed investigation of the results from LMRoe on turbulence has not been conducted yet.

Apart from the numerical complexities of simulating compressible flows at low Mach numbers, one other issue is how to simulate turbulent flows accurately and efficiently under these situations. If the Reynolds number is too high and/or the available numerical resolution is not sufficient, DNS is not an appropriate choice. In such cases, one must use either the Reynolds-averaged Navier–Stokes-equation (RANS) methods or large eddy simulations (LES). The latter is always preferred for simulations of time-evolving turbulent flows. Tajallipour et al. [8] proposed a self-adaptive upwind method for LES to reduce the numerical dissipation from the Roe scheme as much as possible. The concept is based on the fact that, if the intensity of a local wiggle is smaller than a preset value, the Roe upwind dissipation term can be ignored with the numerical scheme remaining stable. The Roe scheme then becomes a central difference scheme to reduce the numerical dissipation. The separation over the airfoil NACA0025 at angles of attack of 0° and 5° can be successfully captured with this method. As in [8], Ciardi et al. [9] also use a local wiggle to determine whether the numerical dissipation can be reduced. Instead of fully turning the Roe upwind dissipation term on or off as in [8], a small value is adopted to gradually increase or decrease the amount of dissipation. According to their result for channel flow at $\mathrm{Re}_\tau = 395$, on a scale between 0 and 1 for the Roe upwind dissipation term, 0.3 is the optimal value. However, this optimal value can only be obtained through trial and error. Kotov et al. [10] modified the flow speed indicator originally developed by Li and Gu [11] for the low-speed Roe scheme to control the numerical dissipation. Based on their modification, the flow speed indicator is determined locally during the calculation, so the amount of numerical dissipation can be provided automatically with less parameter tuning. In summary, all of these results share the same objective, which is to minimize the numerical dissipation as much as possible to prevent adulterating the sub-grid scale (SGS) model while keeping the numerical scheme stable.

Instead of reducing the numerical dissipation and then calculating the SGS model separately to provide an appropriate dissipation such as in [8–10], another way of handling numerical dissipation is to numerically solve the Navier–Stokes equations on a coarse LES, having the truncation error of the numerical scheme serve as a turbulence model, i.e., an implicit LES (ILES). This approach was originally proposed by Boris et al. [12] and reviewed recently in the monograph edited by Margolin and Rider [13]. The ILES methodology is justified based on the practical observation that truncation errors introduce numerical dissipation and its effect is qualitatively similar to the effects of an explicit SGS model. For instance, one can observe a $k^{-5/3}$-energy dependence in numerical simulations by Porter and Woodward [14] of decaying isotropic turbulence performed using an Euler solver. Such results can be used to support the ILES methodology but there are also contradicting results. Garnier et al. [15] analyzed several different shock-capturing Euler schemes applied to decaying isotropic turbulence and found typical behavior associated with low Reynolds number flows rather than that expected from high Reynolds number LES. He determined that ILES provided substantially more numerical dissipation than expected based on the physics of turbulent cascade. Similar conclusions were reached by Domaradzki and Radhakrishnan [16], who showed that the ILES results for rotating and non-rotating turbulence were sensitive to the time step and the method failed to produce theoretically expected results for certain initial conditions and for rotating turbulence. Actually, ILES shares a common feature with all turbulence models; specifically, all of them attempt to dissipate the unphysical energy, which accumulates in small scales because of numerical under-resolution. However, it should be recognized that numerical dissipation produced from the ILES always lacks physical meaning.

Therefore, neither reducing the numerical dissipation then using the SGS model to reintroduce dissipation nor taking advantage of pure numerical dissipation as an implicit turbulence model; the former wastes calculation time for the same purpose and the latter lacks physical meaning. The aim of this study is to propose a new implicit turbulence model in the LES framework for LMRoe. The new concept revolves around whether numerical dissipation in ILES can be appropriately controlled so as to conform to actual energy transfer from resolved to unresolved, subgrid scales, so that ILES gives physically realistic results. With this in mind, the Roe upwind dissipation term is used to remove the unphysical energy at small scales while using the energy ratio (*ER*) [17] as a criterion to automatically and physically adjust the amount of dissipation. Thus, the present turbulence model, called the automatic dissipation adjustment (ADA) model, receives the benefit of ILES, which is that no explicit modeling terms are needed, and also places the numerical dissipation term on a firm physical foundation because *ER* has its roots in the truncated Navier Stokes (TNS) procedure [18]. For the turbulent channel flow a lower Reynolds number of $\mathrm{Re}_\tau = 180$, we show that LMRoe can significantly reduce numerical dissipation to ensure that the Roe upwind dissipation term potentially establishes an implicit turbulence model. However, with the ADA model, the Roe