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A domain integral equation approach for simulating two dimensional transverse electric scattering in a layered medium with a Gabor frame discretization

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We solve the 2D transverse-electrically polarized domain-integral equation in a layered background medium by applying a Gabor frame as a projection method. This algorithm employs both a spatial and a spectral discretization of the electric field and the contrast current in the direction of the layer extent. In the spectral domain we use a representation on the complex plane that avoids the poles and branchcuts found in the Green function. Because of the special choice of the complex-plane path in the spectral domain and because of the choice to use a Gabor frame to represent functions on this path, fast algorithms based on FFTs are available to transform to and from the spectral domain, yielding an $O(N \log N)$ scaling in computation time.

1. Introduction

For several applications in electrical engineering it is vital to have fast and accurate models to calculate the scattering of electromagnetic waves from dielectric structures of finite size. Among these are metrology for integrated circuit production [1,2], various elements on nanophotonic chips [3,4] and metamaterials [5]. For these applications, structures are often embedded in a host medium with multiple layers of different materials.

Many numerical methods have already been developed for the characterization of electromagnetic scattering in a multilayered medium, e.g. local formulations such as finite difference time domain (FDTD) [6] and the Finite Element Method (FEM) [7,8]. Global formulations that employ a Green function exist in both a time-domain formulation and a time-harmonic formulation. In the time domain the Green function can be generalized to multilayered media as well [9–11]. Since we are interested in scattering from monochromatic light-sources, we are interested in a time-harmonic formulation. Such an integral formulation requires solving a nonhomogeneous matrix equation, which can be solved efficiently with an iterative solver, especially when the matrix vector product can be computed rapidly. One popular approach is to speed up this matrix vector product by decomposing the Green function into long-range and short-range interactions, combined with a hierarchical division of the simulation domain. Examples of such methods for a homogeneous medium are the Fast Multipole Method (FMM) [12] and the Fast Inhomogeneous Plane Wave Algorithm (FIPWA) [13,14]. Extensions to multilayer media exists both for FMM [15] and for FIPWA [16]. Another popular approach for fast matrix vector products exploits the

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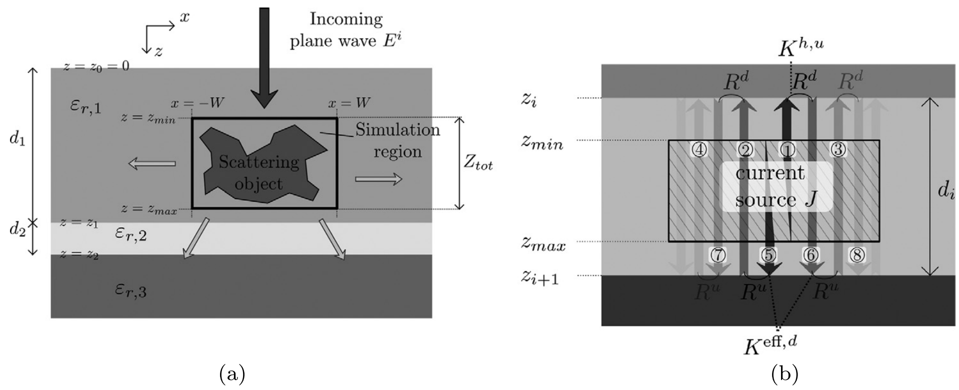

Fig. 1. (a) The scattering setup. (b) The source of reflections including the definition of the different K waves.

observation that the Green function in a layered medium exhibits a translation symmetry in the direction parallel to the layer interfaces. We will focus on a method that exploits this translation symmetry.

A domain integral formulation consists of two parts. The first part is the calculation of the contrast current density from the electric field and the contrast function. The second part is an integral over the product of the Green function and the contrast current density that yields the scattered electric field. For homogeneous media, the spatial Green function is readily obtained. Among the free-space methods that employ the Green function in the spatial domain and exploit the translation symmetry are the Conjugate Gradient Fast Fourier Transform CGFFT [17] and its enhancements, such as the Adaptive Integral Method (AIM) [18] and the pre-corrected FFT [19].

To use similar methods in stratified media, the multi-layered Green function is required. The main differences between the two-dimensional free-space Green function and the two-dimensional stratified-medium Green function are the reflections and transmissions at the layer interfaces in layered media. For stratified media, an exact expression for the Green function in the spectral domain can be derived, so in principle it is possible to calculate the Green function completely in the spatial domain via a Fourier transform. However, calculating the pertaining Fourier integral is far from trivial, since there are branchcuts and poles present in the Green function. Several methods exist to calculate these so-called Sommerfeld integrals e.g. the Discrete Complex Image Method (DCIM) [20], the steepest descent path (SDP) [21], Sommerfeld tail extrapolation [22], and a method based on a perfectly matched layer [23,24].

An alternative approach is to consider the so-called spectral methods, in which the exact spectral-domain Green function is employed directly. For periodically repeating scattering structures, such as optical gratings, several spectral methods have already been developed. Important examples are the Rigorous Coupled Wave Analysis (RCWA), also called the Fourier Modal Method [25,26], and some periodic-volume-integral-equation-based methods (PVIE) [27]. Then, the periodicity can be exploited because the periodicity implies a discrete spectral domain and therefore an obvious and well-performing discretization of the spectral domain exists. These methods can be adapted to solve a-periodic structures as well, for example via perfectly matched layers (PML) [28] or supercell techniques, but even then these solvers are in essence periodic.

Here we present a mixed spatial–spectral method that is completely a-periodic in nature. This also implies that the spectral domain is now continuous instead of discrete. Consequently, branchcuts and poles in the spectral Green function need to be treated carefully. We demonstrate an approach that employs a representation of the fields in the spectral-domain complex plane, that avoids the poles found in the Green function. This representation has been specifically chosen such that the Green functions consist of smooth functions with an effectively limited support, while still allowing for efficient transformations to and from the spectral domain with $O(N \log N)$ computational complexity, for N degrees of freedom. The finite support in both the spectral and the spatial domain allows for a convenient discretization in terms of Gabor frames. Owing to the Gabor frames, all operations are of $O(N \log N)$ complexity or less, thus yielding an $O(N \log N)$ scaling with the number of unknowns.

We start this paper with details about the formulation, after which we present the discretization scheme. Subsequently, the spectral complex-plane path and the representation of the Green functions on this path are illustrated. We conclude with three numerical examples to demonstrate the proposed scheme.

2. Formulation

2.1. Problem definition

Consider a layered medium, i.e. a structure of $N-1$ horizontal layers stacked in the z-direction, with relative permittivities $\varepsilon_{r,n}$ and thicknesses d_n. The space below the stack has permittivity $\varepsilon_{r,N}$ and above the stack has permittivity $\varepsilon_{r,0}$. In layer i a two dimensional dielectric object in the $x-z$ plane is described by the permittivity function $\varepsilon_r(x,z)$. The object is contained within the rectangle $x \in [-W, W]$, $z \in [z_{min}, z_{max}]$, which we call the simulation domain. Fig. 1(a) shows our scattering setup for $N = 3$ and $i = 1$.