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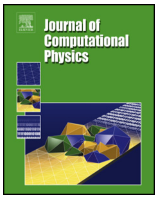

Parallel numerical modeling of hybrid-dimensional compositional non-isothermal Darcy flows in fractured porous media

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ABSTRACT

This paper introduces a new discrete fracture model accounting for non-isothermal compositional multiphase Darcy flows and complex networks of fractures with intersecting, immersed and non-immersed fractures. The so called hybrid-dimensional model using a 2D model in the fractures coupled with a 3D model in the matrix is first derived rigorously starting from the equi-dimensional matrix fracture model. Then, it is discretized using a fully implicit time integration combined with the Vertex Approximate Gradient (VAG) finite volume scheme which is adapted to polyhedral meshes and anisotropic heterogeneous media. The fully coupled systems are assembled and solved in parallel using the Single Program Multiple Data (SPMD) paradigm with one layer of ghost cells. This strategy allows for a local assembly of the discrete systems. An efficient preconditioner is implemented to solve the linear systems at each time step and each Newton type iteration of the simulation. The numerical efficiency of our approach is assessed on different meshes, fracture networks, and physical settings in terms of parallel scalability, nonlinear convergence and linear convergence.

1. Introduction

Flow and transport in fractured porous media are of paramount importance for many applications such as petroleum exploration and production, geological storage of carbon dioxide, hydrogeology, or geothermal energy. Two classes of models, dual continuum and discrete fracture models, are typically employed and possibly coupled to simulate flow and transport in fractured porous media. Dual continuum models assume that the fracture network is well connected and can be homogenized as a continuum coupled to the matrix continuum using transfer functions. On the other hand, discrete fracture models (DFM), on which this paper focuses, represent explicitly the fractures as codimension one surfaces immersed in the surrounding matrix domain. The use of lower dimensional rather than equi-dimensional entities to represent the fractures has been introduced in [1–5] to facilitate the grid generation and to reduce the number of degrees of freedom of the discretized model. The reduction of dimension in the fracture network is obtained from the equi-dimensional model by integration and averaging along the width of each fracture. The resulting so called hybrid-dimensional models couple the 3D model in the matrix with a 2D model in the fracture network taking into account the jump of the normal fluxes as

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well as additional transmission conditions at the matrix fracture interfaces. These transmission conditions depend on the mathematical nature of the equi-dimensional model and on additional physical assumptions. They are typically derived for a single phase Darcy flow for which they specify either the continuity of the pressure in the case of fractures acting as drains (see [1,6]) or Robin type conditions in order to take into account the discontinuity of the pressure for fractures acting as barriers (see [2,5,7,8]). Different transmission conditions are derived in [9] in the case of a linear hyperbolic equation, and in [10–14] in the case of two-phase immiscible Darcy flows.

The discretization of hybrid-dimensional Darcy flow models has been the object of many works. In [4] a cell-centred Finite Volume scheme using a Two Point Flux Approximation (TPFA) is proposed assuming the orthogonality of the mesh and isotropic permeability fields. Cell-centred Finite Volume schemes can be extended to general meshes and anisotropic permeability fields using MultiPoint Flux Approximations (MPFA) (see [15–19]). MPFA schemes can lack robustness on distorted meshes and large anisotropies due to the non-symmetry of the discretization. They are also very expensive compared with nodal discretizations on tetrahedral meshes. In [1], a Mixed Finite Element (MFE) method is proposed for single phase Darcy flows. It is extended to two-phase flows in [11] in an IMPES framework using a Mixed Hybrid Finite Element (MHFE) discretization for the pressure equation and a Discontinuous Galerkin discretization of the saturation equation. The Hybrid Finite Volume and Mimetic finite difference schemes, belonging to the family of Hybrid Mimetic Mixed Methods (HMM) [20], have been extended to hybrid-dimensional models in [21,22] as well as in [6,8] in the more general Gradient Discretization framework [23]. These approaches are adapted to general meshes and anisotropy but require as many degrees of freedom as faces. Control Volume Finite Element Methods (CVFE) [3,10,24,15] have the advantage to use only nodal unknowns leading to much fewer degrees of freedom than MPFA and HMM schemes on tetrahedral meshes. On the other hand, at the matrix fracture interfaces, the control volumes have the drawback to be shared between the matrix and the fractures. It results that a strong refinement of the mesh is needed at these interfaces in the case of large contrasts between the matrix and fracture permeabilities. This article focus on the Vertex Approximate Gradient (VAG) scheme which has been introduced for the discretization of multiphase Darcy flows in [25] and extended to hybrid-dimensional models in [13,6,8,9,14]. The VAG scheme uses nodal and fracture face unknowns in addition to the cell unknowns which can be eliminated without any fill-in. Thanks to its essentially nodal feature, it leads to a sparse discretization on tetrahedral or mainly tetrahedral meshes. It has the advantage, compared with the CVFE methods of [3,10,24] or [26], to avoid the mixing of the control volumes at the matrix fracture interfaces, which is a key feature for its coupling with a transport model. As shown in [13] for two-phase flow problems, this allows for a coarser mesh size at the matrix fracture interface for a given accuracy. Let us also mention that non-matching discretizations of the fracture and matrix meshes are studied for single phase Darcy flows in [27–29] and [30].

The first objective of this paper is to extend the derivation of the hybrid-dimensional model to the case of non-isothermal compositional multiphase Darcy flows. To focus on compositional non-isothermal features, capillary pressures are not considered in this paper. They could be included following the usual phase based upwinding approach as in [25] or recent ideas developed in [31] for two-phase flows. Let us refer to [32] for a comparison of both approaches in the case of an immiscible two-phase flow using a reference solution provided by the equi-dimensional model in the fractures. All the underlying assumptions of our reduced model will be carefully stated. In particular, the fractures are considered as previous and are assumed not to act as barriers. It results, as in [1], that the pressure can be considered as continuous at the matrix fracture interfaces. The hybrid-dimensional model accounts for complex network of fractures including intersecting, immersed and non-immersed fractures. The formulation of the compositional model is based on a Coats' type formulation [33,34] extending the approach presented in [25] to non-isothermal flows. It accounts for an arbitrary nonzero number of components in each phase allowing to model immiscible, partially miscible or fully miscible flows.

The second objective of this paper is to extend the VAG discretization to our model and to develop an efficient parallel algorithm implementing the discrete model. Following [25,13], the discretization is based on a finite volume formulation of the component molar and energy conservation equations. The definition of the control volumes is adapted to the heterogeneities of the porous medium and avoids in particular the mixing of matrix and fracture rocktypes for the degrees of freedom located at the matrix fracture interfaces. The fluxes combine the VAG Darcy and Fourier fluxes with a phase based upwind approximation of the mobilities. A fully implicit Euler time integration coupling the conservation equations with the local closure laws including thermodynamical equilibrium is used in order to avoid severe limitations on the time step due to the high velocities and small control volumes in the fractures.

The discrete model is implemented in parallel based on the SPMD (Single Program, Multiple Data) paradigm. It relies on a distribution of the mesh on the processes with one layer of ghost cells in order to allow for a local assembly of the discrete systems. The key ingredient for the efficiency of the parallel algorithm is the solution, at each time step and at each Newton type iteration, of the large sparse linear system coupling the physical unknowns on the spatial degrees of freedom of the VAG scheme. Our strategy is first based on the elimination, without any fill-in, of both the local closure laws and the cell unknowns. Then, the reduced linear system is solved using a parallel iterative solver preconditioned by a CPR-AMG preconditioner introduced in [35] and [36]. This state of the art preconditioner combines multiplicatively an Algebraic MultiGrid (AMG) preconditioner for a proper pressure block of the linear system with a local incomplete factorization preconditioner for the full system. The numerical efficiency of the algorithm, in terms of parallel scalability, nonlinear convergence and linear convergence, is investigated on several test cases. We consider different families of meshes and different complexity of fracture networks ranging from a few fractures to say about 1000 fractures with highly contrasted matrix fracture permeabilities. The test cases incorporate different physical models including one isothermal immiscible