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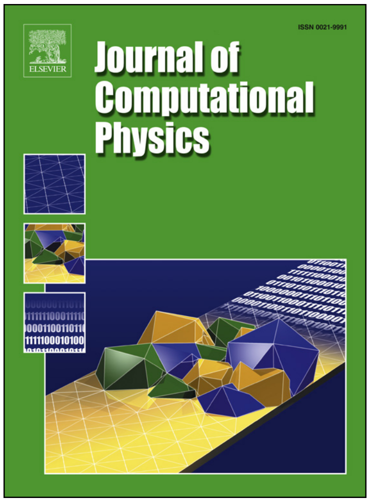

Simulation of incompressible two-phase flow in porous media with large timesteps

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Simulation of incompressible two-phase flow in porous media with large timesteps

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Abstract

Multiphase flow in porous media occurs in several disciplines including petroleum reservoir engineering, petroleum systems' analysis, and CO_2 sequestration. While simulations often use a fully implicit discretization to increase the time step size, restrictions on the time step often exist due to non-convergence of the nonlinear solver (e.g. Newton's method). Here this problem is addressed for the Buckley-Leverett equations, which model incompressible, immiscible, two-phase flow with no capillary potential. The equations are recast as a gradient flow using the phase-field method, and a convex energy splitting scheme is applied to enable large timesteps, even for high degrees of heterogeneity in permeability and viscosity. By using the phase-field formulation as a homotopy map, the underlying hyperbolic flow equations can be solved with large timesteps. For a heterogeneous test problem, the new homotopy method allows the timestep to be increased by more than six orders of magnitude relative to the unmodified equations while maintaining convergence.

Keywords: Reservoir simulation, two-phase flow, phase-field method, homotopy, multigrid

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1. Introduction

Timestep restrictions during simulation of fluid flow through extremely heterogeneous porous media remain a significant limiting factor in petroleum reservoir models [1]. The equations of flow in porous media [2, 3, 4] suffer from severe timestep restrictions as the permeability and viscosity contrast become increasingly heterogeneous [5, 6, 7], even when solved with a fully implicit discretization. Often there is no choice but to significantly reduce the timestep in order to regain convergence. Improved numerical methods have helped [8, 5, 6, 7], but the problem persists because it originates from the shape of the fractional flow function which leads to divergence of the iterative method used to solve the discrete nonlinear equations.

Either the numerical method or the equations themselves must be modified to achieve convergence for large timesteps. Previous efforts have focused on the numerical method with the use of line-search [8], trust-region [5, 7], or continuation methods [6]. Here we focus on regularizing the equations themselves with the addition of an energy constraint. Gradient flows, where evolution equations are derived from energy functionals, offer an attractive alternative formulation for the flow equations. In particular, the phase-field method has emerged as an effective way to solve free boundary problems without explicitly tracking interfaces, and models of fluid flow have been rigorously derived from thermodynamic principles [9, 10, 11, 12]. Phase-field methods offer numerical advantages as well, since they guarantee monotonically decreasing energy of the solution. For example, Feng and Wise [13, 14] recently showed that a Cahn-Hilliard-Darcy system has an unconditionally energy-stable and unconditionally uniquely solvable discretization.

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