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A shock-fitting technique for cell-centered finite volume methods on unstructured dynamic meshes

Dongyang Zou [a,b], Chunguang Xu [a,b], Haibo Dong [a,b], Jun Liu [a,b,*]

[a] State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116024, PR China
[b] School of Aeronautics and Astronautics, Dalian University of Technology, Dalian 116024, PR China

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ABSTRACT

In this work, the shock-fitting technique is further developed on unstructured dynamic meshes. The shock wave is fitted and regarded as a special boundary, whose boundary conditions and boundary speed (shock speed) are determined by solving Rankine–Hugoniot relations. The fitted shock splits the entire computational region into subregions, in which the flows are free from shocks and flow states are solved by a shock-capturing code based on arbitrary Lagrangian–Eulerian algorithm. Along with the motion of the fitted shock, an unstructured dynamic meshes algorithm is used to update the internal node's position to maintain the high quality of computational meshes. The successful applications prove the present shock-fitting to be a valid technique.

1. Introduction

Shock waves occur when the gas is sharply compressed. In all flow fields in which they occur, shock waves play an important role that affects the overall flow behavior. Computing the shock waves correctly represents a difficult problem.

In 1950, von Neumann pioneered the concept of adding sufficient artificial viscosity to the inviscid flow equations to capture the shock [1]. In 1954, Lax proposed the famous "weak solution theory" which solves the governing equations in integral form rather than in differential form [2]. Based on the weak solution theory, various shock-capturing schemes were devised and widely used for solving two dimensional steady flows [3,4]. However, due to a presence of spurious numerical oscillations in the vicinity of the shock waves, applications of high order shock-capturing schemes were greatly limited.

Subsequently, studies of the underlying mechanisms that produce these oscillations have attracted the attention of many scientists [5–7]. These studies proved to be successful and led to algorithms known as oscillation-free schemes. Many capturing algorithms were developed and used, including total variation diminishing (TVD) schemes [8–11], essentially non-oscillatory (ENO) schemes [12], weighted essentially non-oscillatory (WENO) schemes [13], non-oscillatory and non-free-parameter dissipation difference (NND) schemes [14].

Despite several creative mathematical solutions proposed to improve the basic shock-capturing technique, it still failed at what was its initial goal. Due to the inability of high-order capturing schemes to pass correctly information through a discontinuity [15], limiter functions are needed in these shock-capturing schemes. A designed limiter function takes effect in the region with a steep gradient so that a discontinuity is smeared over a few grid points in a shock-capturing algorithm. Therefore, a captured shock relies heavily on the computational mesh. Mesh refinement is a direct way to increase the so-

* Corresponding author at: School of Aeronautics and Astronautics, Dalian University of Technology, Dalian 116024, PR China.
E-mail address: liujun65@dlut.edu.cn (J. Liu).

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lution quality without considering the computational cost. However, the mesh refinement does not reduce the oscillations, rather it decreases the wave length [16]. In other words, the approach to improve solution quality by refining the mesh has not changed the dependency on the use of limiter functions. The situation is worse if the computational mesh is unstructured. This is because there is always at least one face not aligned with the shock on an unstructured mesh. The damage brought about by orthogonality increases the amplitude of these spurious oscillations. Some studies [17,18] on a truly multidimensional or rotational upwind scheme have been applied to eliminate these spurious oscillations. Due to the limitations of the current technology, the studies of multidimensional schemes have not entirely solved the problems of oscillations. As Arora and Roe [19] pointed out, it is difficult to ensure a captured shock to be both narrow and oscillations-free.

As an alternative, the shock-fitting technique is reconsidered to eliminate the disadvantages in dealing with shock waves by the shock-capturing technique. The shock-fitting technique was a reliable tool to simulate a shock wave since the dawn of computational fluid dynamics (CFD). In the 1960s, Moretti et al. [20] developed a shock-fitting technique together with the Lax–Wendroff scheme, which produced a set of solutions of supersonic blunt body flows requiring only 6 minutes on a CDC 6600 computer. This work greatly advanced the development of computational gas dynamics at the time. In 1967, Richtmyer and Morton [21] proposed what was known as floating shock-fitting method, which admitted the fitted shock wave to float on the background mesh. The floating shock-fitting eliminated the problems associated with partitioning the flow field using the boundary-shock implementation. Salas [22] applied the floating-shock fitting technique to simulate flows with internal discontinuities, including shock waves and contact surfaces, which greatly promoted the development of the shock-fitting technique. The floating shock-fitting technique was implemented and combined with a second-order-accurate upwind scheme by Hartwich [23], which makes it possible to construct a general, non-conservative Euler solver. With additional studies, more and more complicated flows were solved by the shock-fitting technique [24]. For example, Nasuti and Onofri used Moretti's shock-fitting technique to compute flows including triple points and shock interactions [25]. In addition, high-order accuracy algorithms were applied with the shock-fitting technique. In 1999, Kopriva [26] combined the shock-fitting technique with the spectral method and applied this method to the computation of supersonic flows. Indeed, the idea of combining the spectral approximation performed only on smooth portions of the flow with a precise description of a shock discontinuity provided by the shock-fitting technique has gained significance for constructing new computational algorithms. In addition, Zhong et al. [27,28] have coupled the high-order finite difference method (FD) with the shock-fitting method and applied it to some hypersonic flows.

Though considerable progress has been achieved with the shock-fitting technique, it has always been unpopular due to a difficult implementation. In terms of coding simplicity, special treatment for discontinuities is not necessary in the shock capturing. To the contrary, in shock-fitting, there are often special points requiring ad hoc treatments, in addition to the topological difficulties that come along with the presence of multiple shocks. The rapid growth of computational resources has given a strong support to CFD. Thus, when computational resources are not limited, most people would prefer a simple algorithm rather than an efficient one.

Researchers practicing shock-fitting proved to be resourceful in search for a viable solution. A shock-fitting technique implemented on unstructured grids has been presented by Paciorri and Bonfiglioli [29,30] and by Bonfiglioli et al. [31]. Compared with the traditional boundary shock-fitting technique on structured grids, the unstructured version of the technique does not suffer from the strong topological limitations that plague boundary shock-fitting implementation on structured grids. In addition, compared with the floating shock-fitting technique, the coupling between shock-fitting algorithms and existing gasdynamic solvers is much simpler for unstructured and structured grids. Salas [32] supported the work based on unstructured grids because of its great potential to generalize the shock-fitting technique in the future.

In the present work, a new method to fit the shock wave is developed based on unstructured dynamic meshes. In the present fitting technique, fitted shocks are treated as discretized boundaries of the computational region. Its motion is driven by the Rankine–Hugoniot (R–H) jump relations. Once all parameters on the shock boundary are determined, fitted shocks are used to move boundaries. In addition, new positions of internal nodes are determined by an unstructured dynamic meshes technique. Along with the motion of the fitted shock, an unstructured dynamic meshes algorithm is used to update the internal nodes' positions to ensure the high quality of the computational mesh.

2. Computational algorithms

In this work, we have combined the shock-fitting algorithm with a previous shock-capturing code [33], which includes the unstructured dynamic meshes algorithm. In order to introduce our work clearly, the new computational code is denoted MCFS. In this new code, the present shock-fitting algorithm is used to determine the shock points' motion and compute the boundary fluxes across each shock boundary face.

We will consider a flow field in which several shock waves occur. As MCFS is used to simulate the flow field, three types of solutions can take place. One of them is the captured solution. In this solution, there is no shock boundary defined in the flow field. By contrast, a fully-fitted solution is obtained when all shock waves in the flow field are fitted as the shock boundary. If some but not all shock waves are fitted, a hybrid solution is obtained.

This section provides a detailed description of the proposed fitting technique along with a brief introduction of the shock-capturing algorithm and unstructured dynamic meshes technique.