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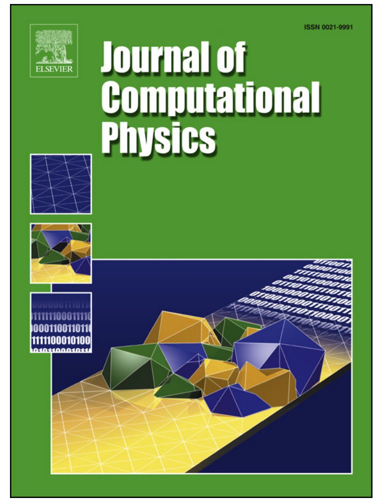

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Partitioned coupling of advection-diffusion-reaction systems and Brinkman flows

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Abstract

We present a partitioned algorithm aimed at extending the capabilities of existing solvers for the simulation of coupled advection-diffusion-reaction systems and incompressible, viscous flow. The space discretisation of the governing equations is based on mixed finite element methods defined on unstructured meshes, whereas the time integration hinges on an operator splitting strategy that exploits the differences in scales between the reaction, advection, and diffusion processes, considering the global system as a number of sequentially linked sets of partial differential, and algebraic equations. The flow solver presents the advantage that all unknowns in the system (here vorticity, velocity, and pressure) can be fully decoupled and thus turn the overall scheme very attractive from the computational perspective. The robustness of the proposed method is illustrated with a series of numerical tests in 2D and 3D, relevant in the modelling of bacterial bioconvection and Boussinesq systems.

Keywords: Advection-reaction-diffusion, viscous flow in porous media, primal-mixed finite element methods, coupling algorithms, operator splitting
2000 MSC: 65M60, 35K57, 76S05, 80A32

1. Introduction

Scope. Our interest is in the efficient solution of advection-diffusion-reaction (ADR) systems coupled with the equations governing incompressible viscous flow within porous media (namely the Stokes-Darcy, or Brinkman equations). A fairly large class of problems in science and engineering assume such a particular structure, as it is one of the basic forms of representing systems where physical, biological, and chemical processes exhibit a remarkable interaction. Notable examples are the density fingering of exothermic fronts in Hele-Shaw cells [19], where hydrodynamic instabilities are strongly influenced by the chemical reactions taking place at different spatial and temporal scales; convection-driven Turing patterns generated using Schnackenberg-Darcy models [26]; reversible reactive flow and viscous fingering in chromatographic separation [2, 29]; plankton dynamics [25]; forced-convective heat and mass transfer in fibrous porous materials [8]; or the bioconvection in porous suspensions of oxytactic bacteria [17, 23]. Phenomena of this kind are also relevant in so-called doubly-diffusive flows [22, 28, 31], where convective effects are driven by two different density gradients having diverse rates of diffusion.

While the specific nature of the physical system of interest will imply diverse forms of coupling mechanisms, our goal is to focusely examine the interaction of the building-block systems through mass transport

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