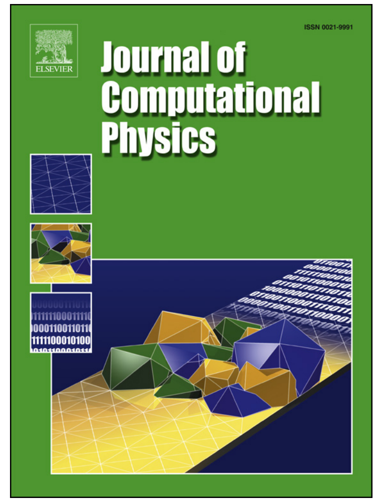

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Hessam Babaee, Minseok Choi, Themistoklis P. Sapsis, George Em Karniadakis

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A robust bi-orthogonal/dynamically-orthogonal method using the covariance pseudo-inverse with application to stochastic flow problems

Hessam Babaee[1,*], Minseok Choi[2], Themistoklis P. Sapsis[1], George Em Karniadakis[2]

[1] *Massachusetts Institute of Technology, Cambridge MA 02139, USA*
[2] *Division of Applied Mathematics, Brown University, Providence, RI 02912, USA*

Abstract

We develop a new robust methodology for the stochastic Navier-Stokes equations based on the dynamically-orthogonal (DO) and bi-orthogonal (BO) methods [1, 2, 3]. Both approaches are variants of a generalized Karhunen-Loève (KL) expansion in which both the stochastic coefficients and the spatial basis evolve according to system dynamics, hence, capturing the low-dimensional structure of the solution. The DO and BO formulations are mathematically equivalent [3], but they exhibit computationally complimentary properties. Specifically, the BO formulation may fail due to crossing of the eigenvalues of the covariance matrix, while both BO and DO become unstable when there is a high condition number of the covariance matrix or zero eigenvalues. To this end, we combine the two methods into a robust hybrid framework and in addition we employ a pseudo-inverse technique to invert the covariance matrix. The robustness of the proposed method stems from addressing the following issues in the DO/BO formulation: (i) eigenvalue crossing: we resolve the issue of eigenvalue crossing in the BO formulation by switching to the DO near eigenvalue crossing using the equivalence theorem and switching back to BO when the distance between eigenvalues is larger than a threshold value; (ii) ill-conditioned covariance matrix: we utilize a pseudo-inverse strategy to invert the covariance matrix; (iii) adaptivity: we utilize an adaptive strategy to add/remove modes to resolve the covariance matrix up to a threshold value. In particular, we introduce a soft-threshold criterion to allow the system to adapt to the newly added/removed mode and therefore avoid repetitive and unnecessary mode addition/removal. When the total variance approaches zero, we show that the DO/BO formulation becomes equivalent to the evolution equation of the Optimally Time-Dependent modes [4]. We demonstrate the capability of the proposed methodology with several numerical examples, namely (i) stochastic Burgers equation: we analyze the performance of the method in the presence of eigenvalue crossing and zero eigenvalues; (ii) stochastic Kovasznay flow: we examine the method in the presence of a singular covariance matrix; and (iii) we examine the adaptivity of the method for an incompressible flow over a cylinder where for large stochastic forcing thirteen DO/BO modes are active.

Keywords: stochastic Navier-Stokes equations; uncertainty quantification; reduced order modeling; dynamical orthogonality; bi-orthogonality.

1. Introduction

Recently, there has been a growing interest in quantifying parametric uncertainty in physical and engineering problems through the probabilistic framework. Such problems are often described by Stochastic Partial Differen-

*Corresponding author.
Email address: hbabaee@pitt.edu (Hessam Babaee)