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3D computation of non-linear eddy currents: Variational method and superconducting cubic bulk

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ABSTRACT

Computing the electric eddy currents in non-linear materials, such as superconductors, is not straightforward. The design of superconducting magnets and power applications needs electromagnetic computer modeling, being in many cases a three-dimensional (3D) problem. Since 3D problems require high computing times, novel time-efficient modeling tools are highly desirable. This article presents a novel computing modeling method based on a variational principle. The self-programmed implementation uses an original minimization method, which divides the sample into sectors. This speeds-up the computations with no loss of accuracy, while enabling efficient parallelization. This method could also be applied to model transients in linear materials or networks of non-linear electrical elements. As example, we analyze the magnetization currents of a cubic superconductor. This 3D situation remains unknown, in spite of the fact that it is often met in material characterization and bulk applications. We found that below the penetration field and in part of the sample, current flux lines are not rectangular and significantly bend in the direction parallel to the applied field. In conclusion, the presented numerical method is able to time-efficiently solve fully 3D situations without loss of accuracy.

1. Introduction

Electrical eddy currents appear in conductors under varying magnetic fields, including the case of wires under alternating currents (AC) of sufficiently high frequency. In certain materials, such as superconductors, the resistivity is highly non-linear, and hence computing their response is not straightforward already in the quasi-magnetostatic situation [1].

Superconductors have been applied to magnet technology for decades and are promising for power applications, such as cables, fault-current limiters, transformers, generators, motors and levitations systems. An important issue of the design of these applications is the electromagnetic response under slowly changing magnetic fields or currents, usually for frequencies below 1 kHz. This design can only be done with computer modeling. In many cases, the situation of study is essentially a three dimensional (3D) problem [1], which involve time-extensive computations. Therefore, novel time-efficient 3D modeling tools are highly desirable.

Regarding material science, the magnetization currents in many situations is 3D, such as bulks shaped as rectanglar prisms, multi-granular samples, and multi-filamentary tapes with a conducting matrix. 3D modeling may also enlighten macroscopic flux cutting effects in the force-free configuration [2,3].

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There are several published 3D modeling results for the finite-element method (FEM) in the following formulations: **H** [4–10], $\mathbf{A}-\phi$ [11–16], $\mathbf{T}-\Omega$ [11], and **H** with cohomology decomposition [9]; being **H** the magnetic field, **A** and ϕ the vector and scalar potentials, and **T** and Ω the current and magnetic potentials. All these approaches require solving the electromagnetic quantities at both the sample volume and surrounding air, setting boundary conditions far away from the sample. Then, only a portion of the degrees of freedom (DoF) are in the sample volume.

The DoF can be greatly reduced by methods taking the current density as state variable, since only the sample volume is taken into account. For mathematically 2D problems, this has been done by the variational method in **J** formulation [17–27], integral methods [28–34] and circuit methods [35]. The boundary-element/finite-element (BEM-FEM) method also avoids meshing the air [36,37]. The FEM integral approach in the $\mathbf{T}-\Omega$ formulation has been reduced to the sample region for 2D cross-sectional problems [38] and 2D surfaces with 3D bending [39–41].

A possible variational method in 3D is very promising. The variational method in the Minimum Electro-Magnetic Entropy Production (MEMEP) implementation has been shown to be highly time efficient, presenting computing times scaling with only power 2 of the number of elements and being able to solve problems in 2D with up to half million DoF in the superconductor [42]. Bossavit introduced the variational method in the **H** formulation in 3D [43], but did not solve any 3D example. Elliott and Kashima provided further insight of the **H** formulation, proposing a mixed formulation of magnetic field and magnetic potential [44,45] and solved simple 3D examples. Prigozhin developed the **J** formulation for 2D surfaces and cross-sectional problems [17–19], which avoids taking DoF in the air. Badia and Lopez found that the functional minimizes the entropy production and introduced the Euler–Lagrange formalism [46,47]. Independently, Sanchez and Navau obtained a method to solve the Critical-State Model (CSM) in cylinders by minimization of a certain magnetostatic energy [48]. However, superconductors in the CSM only minimize the magnetostatic energy in the initial curve from zero-field cool and special situations [21,49], being that method not applicable for arbitrarily non-uniform applied fields, arbitrary cross-sections, or simultaneous transport current and applied field, such as in a coil. In any case, the involved mechanisms are irreversible. A 3D variational principle in the $\mathbf{J}-\phi$ (or $\mathbf{J}-q$, where q is the charge density) formalism was obtained in [23,50]. Except for axi-symmetrical or infinitely long shapes, that method needs to compute **J** and q iteratively, which increases the computing time [50].

Independently on the numerical method, the magnetization currents in rectangular prisms of finite thickness remains mostly unknown, being a cube a particular case of this shape. Infinite rectangular prisms in the CSM were analytically solved in [51]. Thin rectangular films have been studied in [28,29] and [19,52] for an isotropic power-law $\mathbf{E}(\mathbf{J})$ relation and the CSM, respectively. Computations for a rectangular prism with a hole has been published in [4] for a power-law $\mathbf{E}(\mathbf{J})$ relation. Reference [53] presented approximated solutions for a cube in the CSM, assuming square current paths. The trapped field of an array of rectangular prisms is computed in [6]. Elliott and Kashima solved a rectangular prism [44] and a sphere under rotating applied field [45], although these works practically do not discuss the results.

This article presents a time-efficient 3D modeling tool based on a variational principle. This modeling tool for non-linear conductors is also efficient to compute transients in linear materials. It could also be easily adapted to modeling the response of networks of many non-linear electrical elements, such as diodes. As a computation example, we analyze a cubic bulk superconductor. We present the model in section 2. Section 2.2 details the deduction of a 3D variational principle in the **T** formulation, which avoids spending DoF in the air and does not require solving the scalar potential or the charge density. The formalism also allows transport currents, in addition to the applied magnetic field. Although here we take an isotropic $\mathbf{E}(\mathbf{J})$ relation into account, the method also allows anisotropic $\mathbf{E}(\mathbf{J})$ relations, such as that for the force-free situation [54]. Our self-programmed implementation uses a non-standard minimization method (section 2.4). This method has been greatly sped up with no loss of accuracy thanks to dividing the sample into sectors, which also enables efficient parallelization (section 2.5). The model is tested by comparing to analytical limits, showing good agreement (section 3). Afterwards, we analyze the superconducting cube for both constant critical-current-density, J_c (section 4.1) and magnetic-field-dependent J_c (section 4.2). The appendices present details of variational calculus of functionals with double volume integrals (Appendix A) and the discretization (Appendix B).

Part of the results of this work have been presented in international conferences in 2015 and 2016 [55,56], the mid-term report of M Kapolka PhD thesis [57] and benchmark 5 of the HTS modeling workgroup [58].

2. Model

In this section, we present the physical assumptions (section 2.1), the variational principle (section 2.2) and several aspects regarding the numerical method and implementation (sections 2.3–2.7).

2.1. Material properties and physical situation

Although the numerical method is valid for any vector $\mathbf{E}(\mathbf{J})$ relation of the material, either isotropic or not, in this work we consider an isotropic power law as

$$\mathbf{E}(\mathbf{J}) = E_c \left(\frac{|\mathbf{J}|}{J_c} \right)^n \frac{\mathbf{J}}{|\mathbf{J}|}, \tag{1}$$