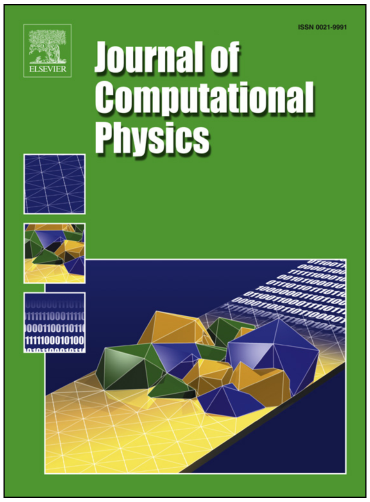

A vertex-centered and positivity-preserving scheme for anisotropic diffusion problems on arbitrary polygonal grids

Xiaoping Zhang, Shuai Su, Jiming Wu

PII: S0021-9991(17)30358-3
DOI: http://dx.doi.org/10.1016/j.jcp.2017.04.070
Reference: YJCPH 7336

To appear in: *Journal of Computational Physics*

Received date: 15 December 2016
Revised date: 5 April 2017
Accepted date: 27 April 2017

Please cite this article in press as: X. Zhang et al., A vertex-centered and positivity-preserving scheme for anisotropic diffusion problems on arbitrary polygonal grids, *J. Comput. Phys.* (2017), http://dx.doi.org/10.1016/j.jcp.2017.04.070

A vertex-centered and positivity-preserving scheme for anisotropic diffusion problems on arbitrary polygonal grids ☆

Xiaoping Zhang[a], Shuai Su[b], Jiming Wu[c,*]

[a] *School of Mathematics and Statistics, Wuhan University, Wuhan, Hubei 430072, P.R. China*
[b] *Graduate School of China Academy of Engineering Physics, Beijing, 100088*
[c] *Institute of Applied Physics and Computational Mathematics, Beijing 100088, P.R. China*

Abstract

We suggest a new positivity-preserving finite volume scheme for anisotropic diffusion problems on arbitrary polygonal grids. The scheme has vertex-centered, edge-midpoint and cell-centered unknowns. The vertex-centered unknowns are primary and have finite volume equations associated with them. The edge-midpoint and cell-centered unknowns are treated as auxiliary ones and are interpolated by the primary unknowns, which makes the final scheme a pure vertex-centered one. Unlike most existing positivity-preserving schemes, the construction of the scheme is based on a special nonlinear two-point flux approximation that has a fixed stencil and does not require the convex decomposition of the co-normal. In order to solve efficiently the nonlinear systems resulting from the nonlinear scheme, Picard method and its Anderson acceleration are discussed. Numerical experiments demonstrate the second-order accuracy and well positivity of the solution for heterogeneous and anisotropic problems on severely distorted grids. The high efficiency of the Anderson acceleration is also shown on reduction of the number of nonlinear iterations. Moreover, the proposed scheme does not have the so-called numerical heat-barrier issue suffered by most existing cell-centered and hybrid schemes. However, further improvements have to be made if the solution is very close to the machine precision and the mesh distortion is very severe.

Keywords: diffusion equation, vertex-centered scheme, positivity-preserving, nonlinear two-point flux approximation

---

1. Introduction

We consider the following diffusion problem

$$-\mathrm{div}(\Lambda\nabla u) = f \quad \text{in} \quad \Omega, \tag{1}$$

$$u = g_D \quad \text{on} \quad \Gamma_D, \tag{2}$$

$$-\Lambda\nabla u\cdot\boldsymbol{n} = g_N \quad \text{on} \quad \Gamma_N, \tag{3}$$

where

☆This work was supported by the National Natural Science Fund of China (Nos. 91330205, 11671313, 11571226).

*Corresponding author

Email addresses: `xpzhang.math@whu.edu.cn` (Xiaoping Zhang), `Sushuaiby@sina.cn` (Shuai Su), `wu_jiming@iapcm.ac.cn` (Jiming Wu)