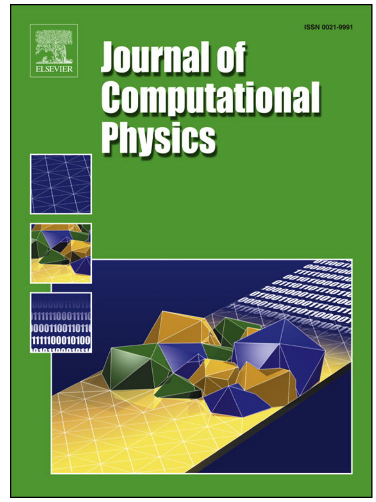
Accepted Manuscript



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PII: S0021-9991(17)30360-1
DOI: http://dx.doi.org/10.1016/j.jcp.2017.04.072
Reference: YJCPH 7338

To appear in: *Journal of Computational Physics*

Received date: 13 April 2017
Accepted date: 27 April 2017

On the relation between conservation and dual consistency for summation-by-parts schemes

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1. Introduction

We consider initial boundary value problems discretized by summation-by-parts (SBP) operators together with the simultaneous approximation term (SAT) technique. In [1, 2, 3, 4], it was shown that dual consistent discretizations on SBP-SAT form, leads to superconvergent linear functionals. For multi-block/element SBP-SAT based schemes [5, 6, 7, 8, 9], conservation is one of the most important issues. In this note, we show that dual consistency and conservation are equivalent concepts for linear conservation laws.

2. The continuous problem

Consider the two coupled linear conservation laws

$$\begin{aligned} u_t + f_L(u)_x &= 0, & -1 \leq x \leq 0,\ \ t > 0, \\ v_t + f_R(v)_x &= 0, & 0 \leq x \leq 1,\ \ t > 0, \\ f_L(u(0,t)) &= f_R(v(0,t)), & x = 0,\ \ t > 0, \end{aligned} \tag{1}$$

augmented with initial conditions. The m component long solution vectors u and v are smooth and continuous, also across the interface. The subscripts L, R refer to the left and right spatial intervals, respectively. For clarity and ease of presentation, we ignore the boundary conditions at $x = \pm 1$.

The coupled problem (1) can be written more compactly as

$$\begin{aligned} w_t + F(w)_x &= 0, & -1 \leq x \leq 1,\ \ t > 0, \\ f_L(u(0,t)) &= f_R(v(0,t)), & x = 0,\ \ t > 0, \end{aligned} \tag{2}$$

where the flux is given by

$$F(w) = \begin{cases} f_L(u), & -1 \leq x \leq 0 \\ f_R(v), & 0 \leq x \leq 1, \end{cases} \quad \text{and} \quad w = \begin{cases} u, & -1 \leq x \leq 0 \\ v, & 0 \leq x \leq 1. \end{cases}$$

For future reference we define the inner product as $(g,h)_a^b = \int_a^b g^T h \, dx$.