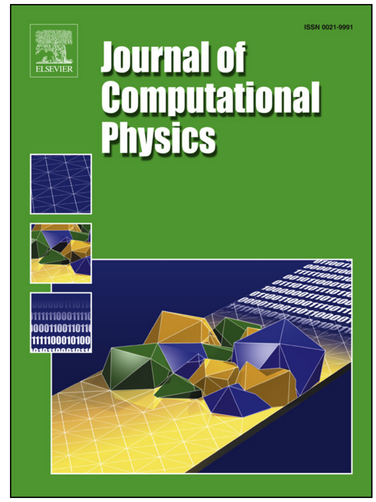
Accepted Manuscript



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PII: S0021-9991(17)30357-1
DOI: http://dx.doi.org/10.1016/j.jcp.2017.04.069
Reference: YJCPH 7335

To appear in: *Journal of Computational Physics*

Received date: 21 September 2016
Revised date: 6 February 2017
Accepted date: 27 April 2017

A new extrapolation cascadic multigrid method for three dimensional elliptic boundary value problems

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Abstract

In this paper, we develop a new extrapolation cascadic multigrid method, which makes it possible to solve three dimensional elliptic boundary value problems with over 100 million unknowns on a desktop computer in half a minute. First, by combining Richardson extrapolation and quadratic finite element (FE) interpolation for the numerical solutions on two-level of grids (current and previous grids), we provide a quite good initial guess for the iterative solution on the next finer grid, which is a third-order approximation to the FE solution. And the resulting large linear system from the FE discretization is then solved by the Jacobi-preconditioned conjugate gradient (JCG) method with the obtained initial guess. Additionally, instead of performing a fixed number of iterations as used in existing cascadic multigrid methods, a relative residual tolerance is introduced in the JCG solver, which enables us to obtain conveniently the numerical solution with the desired accuracy. Moreover, a simple method based on the midpoint extrapolation formula is proposed to achieve higher-order accuracy on the finest grid cheaply and directly. Test results from four examples including two smooth problems with both constant and variable coefficients, an H^3-regular problem as well as an anisotropic problem are reported to show that the proposed method has much better efficiency compared to the classical V-cycle and W-cycle multigrid methods. Finally, we present the reason why our method is highly efficient for solving these elliptic problems.

Keywords: Richardson extrapolation, cascadic multigrid method, elliptic equation, quadratic FE interpolation, high efficiency
2000 MSC: 65N06, 65N55

1. Introduction

Elliptic boundary value problems arise in many physical problems. Consider the following three dimensional (3D) elliptic problem:

$$\begin{cases} -\nabla \cdot (\beta(\mathbf{x})\nabla u) = & f(\mathbf{x}) \quad \text{in } \Omega, \\ u = & g_D(\mathbf{x}) \quad \text{on } \Gamma_D, \\ \alpha(\mathbf{x})u + \beta(\mathbf{x})\dfrac{\partial u}{\partial n} = & g_R(\mathbf{x}) \quad \text{on } \Gamma_R, \end{cases} \tag{1}$$

where α and β are smooth functions on $\overline{\Omega}$ and $0 < \beta_{min} \leq \beta \leq \beta_{max}$ for every $\mathbf{x} \in \Omega$, $f : \Omega \to \Re$, $g_D : \Gamma_D \to \Re$ and $g_R : \Gamma_R \to \Re$ are assigned functions. Here Ω is a bounded convex domain in R^3 with Dirichlet boundary Γ_D and

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