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Computational reduction strategies for the detection of steady bifurcations in incompressible fluid-dynamics: Applications to Coanda effect in cardiology

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ABSTRACT

We focus on reducing the computational costs associated with the hydrodynamic stability of solutions of the incompressible Navier–Stokes equations for a Newtonian and viscous fluid in contraction–expansion channels. In particular, we are interested in studying steady bifurcations, occurring when non-unique stable solutions appear as physical and/or geometric control parameters are varied. The formulation of the stability problem requires solving an eigenvalue problem for a partial differential operator. An alternative to this approach is the direct simulation of the flow to characterize the asymptotic behavior of the solution. Both approaches can be extremely expensive in terms of computational time. We propose to apply Reduced Order Modeling (ROM) techniques to reduce the demanding computational costs associated with the detection of a type of steady bifurcations in fluid dynamics. The application that motivated the present study is the onset of asymmetries (i.e., symmetry breaking bifurcation) in blood flow through a regurgitant mitral valve, depending on the Reynolds number and the regurgitant mitral valve orifice shape.

1. Introduction

In this work we study the hydrodynamic stability of solutions of the incompressible Navier–Stokes equations for a Newtonian and viscous fluid in contraction–expansion channels, with a particular concern on steady bifurcations. Steady bifurcations occur when new, non-unique solution branches of the Navier–Stokes equations appear as physical and/or geometric control parameters are varied. When the fluid domain is characterized by two or three dimensions with non-periodic boundary conditions, the formulation of the stability problem requires solving an eigenvalue problem for a partial differential operator. See [1] for a review on numerical methods for stability analysis based on linearized eigenvalue problems. An alternative to the eigenvalue problem approach is the direct simulation of the flow to characterize the asymptotic behavior of the solution; see, e.g., [2–4]. Both approaches can be extremely expensive in terms of computational time. In this paper, we propose to apply Reduced Order Modeling (ROM) techniques to reduce the demanding computational costs associated with flow stability analysis.

Practical applications of contraction–expansion channel flows include equipments such as heat exchangers, combustion chambers, and mixing vessel. An application that motivated the present study is the onset of asymmetries (i.e., symmetry

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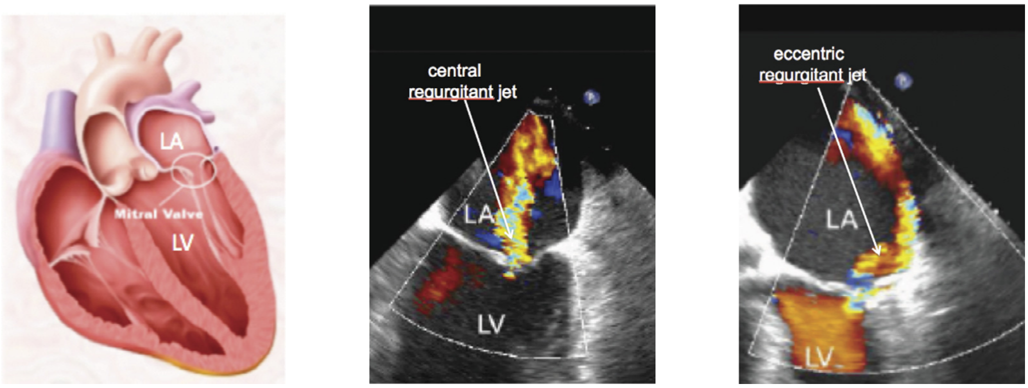

(a) Location of Mitral Valve (b) Central color Doppler jet (c) Eccentric color Doppler jet

Fig. 1. (a) Anatomy of the heart showing the mitral valve. (b) Echocardiographic image of central regurgitant jet flowing from the left ventricle (LV) to the left atrium (LA). Colors denote different fluid velocities. (c) Echocardiographic image of eccentric regurgitant jet, hugging the walls of the left atrium (LA) known as the Coanda effect.

breaking bifurcation) in blood flow through a regurgitant mitral valve, depending on the Reynolds number and the regurgitant mitral valve orifice shape. Mitral regurgitation is a valvular disease characterized by abnormal leaking of blood through the mitral valve from the left ventricle into the left atrium of the heart. See Fig. 1. In certain cases the regurgitant jet "hugs" the wall of the heart's atrium as shown in Fig. 1(c). These eccentric, wall-hugging, non-symmetric regurgitant jets have been observed at low Reynolds numbers [5,6] and are said to undergo the Coanda effect [7,8]. This effect, described as the tendency of a fluid jet to be attracted to a nearby surface, owes its name to Romanian aerodynamics pioneer Henri Coanda. The primary tool to assess the severity of mitral regurgitation is echocardiography [9]. One of the biggest challenges in echocardiographic assessment of mitral regurgitation is the Coanda effect: the wall-hugging jets appear smaller in the color Doppler image of regurgitant flow, leading to a gross under-estimation of regurgitant volume by inexperienced observers [10,11]. As a result, patients requiring treatment may not be recognized.

Despite the large cardiovascular and bioengineering literature reporting on the Coanda effect in echocardiographic assessment of mitral regurgitation, there is very little connection with the fluid dynamics literature that could help identify and understand the main features of the corresponding flow conditions. In this paper, our goal is to understand what triggers the Coanda effect in a simplified setting. A contraction–expansion channel is a simplified setting which has the same geometric features of mitral regurgitation. In fact, a mitral regurgitant jet flows from the left ventricle through the contraction between the mitral leaflet, called regurgitant orifice, into the left atrium. First, we focus on planar contraction–expansion channels (see Fig. 2) and investigate the influence of the Reynolds number and the contraction width w_c (i.e., the orifice height) on the flow. Then, we consider the 3D geometry reported in Fig. 3 to understand the role played by the channel depth h (i.e., the orifice length). Eccentric regurgitant jets typically occur in prolapsed mitral valves, i.e. when two valve flaps of the mitral valve do not close evenly. Thus, another parameter of interest, although not considered in this work, could be the orifice depth. Moreover, for a more realistic setting one would have to account for the pulsatility of the flow and include the Strouhal number among the parameters.

We remark that the focus of this paper is to investigate the cause of the Coanda effect in simplified settings. Nonetheless, it is thanks to the results reported here that our medical collaborators at the Houston Methodist DeBakey Heart & Vascular Center were able to reproduce the Coanda effect in a mock heart chamber (see Sec. 4.4). A comparison between the experiments in vitro and corresponding 3D simulations is presented in [12].

The incompressible fluid dynamics in a planar contraction–expansion channel has been widely studied from both theoretical and practical perspectives; see, e.g., [13–18] and references therein. In the two-dimensional geometry reported in Fig. 2, the wall-hugging effect happens only above a critical Reynolds number (12), which depends on the expansion ratio λ defined in (11). Compare Fig. 5(e) and Fig. 5(b), which correspond to a Reynolds number above and below the critical value, respectively. The asymmetric, wall-hugging solution remains stable for a certain range of Reynolds number and asymmetries become stronger with the increasing Reynolds number, as shown in [18]. The formation of stable asymmetric vortices in 2D planar expansion is attributed to an increase in velocity near one wall that leads to a decrease in pressure near that wall [8]. Once a pressure difference is established across the channel, it will maintain the asymmetry of the flow. The critical value of the Reynolds number has been identified for different expansion ratios λ. In particular, it was found that such critical value decreases with increasing value of λ (see [14,19]).

In the three-dimensional geometry reported in Fig. 3, the critical Reynolds number for the symmetry-breaking (i.e., the wall-hugging) varies with the expansion ratio and the aspect ratio defined in (13), as shown in [20–22]. When the expansion ratio is fixed and the aspect ratio decreases, the endwall influence becomes more important: the critical Reynolds number increases [21,22]. For moderate aspect ratios, the flow is steady in time but highly three-dimensional, and complex spiraling structures are observed, which are not closed recirculating cells as in the case of 2D flows. See Fig. 15. The numerical studies