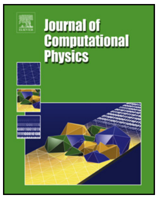
Journal of Computational Physics 344 (2017) 623–646

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp



Efficient solution of 3D electromagnetic eddy-current problems within the finite volume framework of *OpenFOAM*

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ARTICLE INFO

Article history:
Received 8 December 2016
Accepted 5 May 2017
Available online 10 May 2017

Keywords:
Eddy-currents
Induction processing
Potential formulation
Maxwell's equations
Finite volume method
Block-coupling
OpenFOAM
Foam-extend

ABSTRACT

Eddy-current problems occur in a wide range of industrial and metallurgical applications where conducting material is processed inductively. Motivated by realising coupled multi-physics simulations, we present a new method for the solution of such problems in the finite volume framework of *foam-extend*, an extended version of the very popular *OpenFOAM* software. The numerical procedure involves a semi-coupled multi-mesh approach to solve Maxwell's equations for non-magnetic materials by means of the Coulomb gauged magnetic vector potential $\boldsymbol{A}$ and the electric scalar potential ϕ. The concept is further extended on the basis of the impressed and reduced magnetic vector potential and its usage in accordance with Biot–Savart's law to achieve a very efficient overall modelling even for complex three-dimensional geometries. Moreover, we present a special discretisation scheme to account for possible discontinuities in the electrical conductivity. To complement our numerical method, an extensive validation is completing the paper, which provides insight into the behaviour and the potential of our approach.

1. Introduction

Eddy-current problems can be found in a wide range of industrial and metallurgical applications. The basic idea of such processes is to use alternating electromagnetic fields, originating from an inductor like a powered coil, to excite eddy-currents in electrically conducting material. Such induced currents produce secondary effects like electromagnetic forces and heat. Depending on the arrangement and geometry of one or more excitation coils in the proximity of the conductor, various different force fields may be tailored for special tasks. The spectrum of possibilities is further extended by the time-dependent behaviour of the driving source current density. Electromagnetic forces are mainly used for the processing of liquid materials e.g. for stirring, mixing, levitation or retention. The scope of possible applications for electromagnetic heat sources is versatile, too. This comprises processes like welding, hardening, melting or casting.

The design of induction processing applications and its electromagnetic effects is often very difficult based only on experimental investigation or measurement. Insights from numerically modelling of eddy-current problems are thus very desirable. Especially in the field of liquid metal and semiconductor processing, most industrial applications rely on a complex interaction of hydrodynamic and electromagnetic effects. Covering multiple physical effects and their interaction is however challenging for numerical models and computer simulations, particularly in three-dimensional space.

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http://dx.doi.org/10.1016/j.jcp.2017.05.005
0021-9991/

Electromagnetic phenomena are most commonly formulated and solved using the finite element method (FEM). Numerous different formulations with its own characteristics exist based on primary and secondary variables [1–5]. The physics is governed by the time-dependent Maxwell's equations [6], which are defined on an unbounded domain. Depending on the application, more or less additional simplifications may be recognised. We will concentrate only on highly conducting, non-magnetic materials, such as non-ferrous metals or semi-conductors at high temperatures. The unboundedness of Maxwell's equations is approximately captured by means of a sufficiently large computational domain.

For many industrial induction processes involving liquids, alternating magnetic fields are used with oscillation frequencies of 1 kHz and above. In those cases, solving a time-dependent electromagnetic problem may not be convenient as very small time-scales have to be resolved, while the time-scales of the coupled phenomena like fluid dynamics or thermodynamics may be independent. If the difference in the order of magnitude of the time-scales is sufficient, a quasi-steady description of Maxwell's equations is advantageous.

In computational fluid dynamics (CFD), the finite volume method (FVM) is the favourable solution in contrast to FEM due to its conservative property [7]. Even though a combination of FEM and FVM for coupled magnetohydrodynamic (MHD) applications is possible [8], its realisation may suffer from reduced efficiency due to additional overhead. This is especially true for simulations with involved, time-dependent geometric changes, where recurring interpolation and grid generation may become a limiting factor. Staying in one single framework, either FEM or FVM, avoids such overhead.

Our claim is to propose a method to solve three-dimensional eddy-current problems based on unstructured, polyhedral FVM, which can be implemented and combined readily within existing CFD-software. This development is motivated by our recent investigation of free-surface flows under the influence of electromagnetic forces in the context of the Ribbon Growth on Substrate (RGS) process [9,10].

The main difficulty thereby is that for unstructured FVM, the size of the computational stencil is limited. Compact numerical stencils however conflict with a proper implicit discretisation of differential operators like $\nabla(\nabla\cdot())$ or $\nabla\times()$. Hence, a suitable formulation of the electromagnetic problem should preferably rely on differential operators which are typical for CFD and known from the Navier–Stokes-Equations [7]. We therefore use a description of Maxwell's equations based on the Coulomb gauged magnetic vector potential $\boldsymbol{A}$ and the electric scalar potential ϕ [5]. An alternative formulation can be found in Djambazov et al. [11]. A closer look however reveals that their implementation is not feasible for very large problems.

In the complex, quasi-steady formulation of the electromagnetic problem, the resulting governing equation system poses an additional challenge for a solution within a finite volume framework. A fully coupled approach of the whole system for a similar, geophysical problem was presented in [12,13] for simple, structured meshes. Even though it is possible to solve the whole system fully coupled, it is not only difficult to implement for unstructured meshes, but it will also require tremendous amounts of memory.

The CFD code *foam-extend* [14], an extended version of the very popular *OpenFOAM* [15–17], provides a special framework for the solution of coupled problems. The implementation for coupled equations is based on block-matrices, which use tensor-valued matrix coefficients. The development in this field is still very active (cf. [18,19]) and is motivated mainly to address the pressure-velocity coupling originating from the Navier–Stokes equations.

To avoid the problems of a fully coupled discretisation, we propose a semi-coupled approach in *foam-extend* which treats the weak coupling between $\boldsymbol{A}$ and ϕ explicitly in a segregated manner (similar to the SIMPLE algorithm [20,7]), while the strong coupling between the complex components of the phasor amplitude is addressed implicitly. The partially block-coupled solution renders techniques like source term linearisation (cf. [11]) obsolete and is much more robust at higher frequencies. Our semi-coupled proposal relies on the discretisation of a non-conducting region around the conducting region of interest. To maintain an effective method, a special multi-mesh implementation has been developed, where two overlapping finite volume meshes are being used.

We will extend our proposal on the basis of the impressed and reduced magnetic vector potential and its usage in accordance with Biot–Savart's law to achieve a very efficient overall modelling even for complex three-dimensional geometries. Although the whole idea of this step is not new and has been used extensively in literature [3,4,21,22] within different finite element frameworks, it is particularly helpful within the finite volume framework. The reason for this is simply that high quality, unstructured finite volume meshes of complex geometries are much more difficult to create than, for example, finite element meshes. Mesh skewness and non-orthogonality may degrade results as only compact numerical stencils are used.

The application of inductors modelled on the basis of Biot–Savart's law offers a great potential to reduce the size and geometrical complexity of the non-conducting region to a minimum, but it comes in general at high computational costs. In our solution concept, this application is very limited, not intended to be used on bulk volume data, and it is also not involved in any iterative steps. Even though we have to deal with a simplified non-conducting region, we expect our proposal to be much faster compared to e.g. [11], where Biot–Savart is iteratively used in a fully segregated approach.

With the formulation of the electromagnetic problem based on the magnetic vector potential $\boldsymbol{A}$, charge conservation of induced currents is not intrinsically satisfied. In fact, the electric scalar potential ϕ, more specifically its gradient, is adjusted to achieve a solenoidal current density field based on a derived Poisson-type differential equation. The corresponding equation is strictly valid only for a continuous electrical conductivity. For cases with large jumps in the electrical conductivity, e.g. at boundaries of different materials, the conducting region has to be either split in several sub-regions or special care has to be taken during discretisation. In order to avoid additional regions with possibly complex geometries and to keep the