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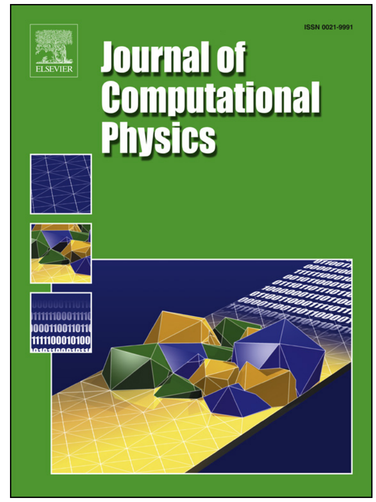

Implicit mesh discontinuous Galerkin methods and interfacial gauge methods for high-order accurate interface dynamics, with applications to surface tension dynamics, rigid body fluid-structure interaction, and free surface flow: Part II

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Implicit mesh discontinuous Galerkin methods and interfacial gauge methods for high-order accurate interface dynamics,
with applications to surface tension dynamics, rigid body fluid-structure interaction, and free surface flow: Part II

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Abstract. In this two-part paper, a high-order accurate implicit mesh discontinuous Galerkin (dG) framework is developed for fluid interface dynamics, facilitating precise computation of interfacial fluid flow in evolving geometries. The framework uses implicitly defined meshes—wherein a reference quadtree or octree grid is combined with an implicit representation of evolving interfaces and moving domain boundaries—and allows physically prescribed interfacial jump conditions to be imposed or captured with high-order accuracy. Part one discusses the design of the framework, including: (i) high-order quadrature for implicitly defined elements and faces; (ii) high-order accurate discretisation of scalar and vector-valued elliptic partial differential equations with interfacial jumps in ellipticity coefficient, leading to optimal-order accuracy in the maximum norm and discrete linear systems that are symmetric positive (semi)definite; (iii) the design of incompressible fluid flow projection operators, which except for the influence of small penalty parameters, are discretely idempotent; and (iv) the design of geometric multigrid methods for elliptic interface problems on implicitly defined meshes and their use as preconditioners for the conjugate gradient method. Also discussed is a variety of aspects relating to moving interfaces, including: (v) dG discretisations of the level set method on implicitly defined meshes; (vi) transferring state between evolving implicit meshes; (vii) preserving mesh topology to accurately compute temporal derivatives; (viii) high-order accurate reinitialisation of level set functions; and (ix) the integration of adaptive mesh refinement.

In part two, several applications of the implicit mesh dG framework in two and three dimensions are presented, including examples of single phase flow in nontrivial geometry, surface tension-driven two phase flow with phase-dependent fluid density and viscosity, rigid body fluid-structure interaction, and free surface flow. A class of techniques known as interfacial gauge methods is adopted to solve the corresponding incompressible Navier-Stokes equations, which, compared to archetypical projection methods, have a weaker coupling between fluid velocity, pressure, and interface position, and allow high-order accurate numerical methods to be developed more easily. Convergence analyses conducted throughout the work demonstrate high-order accuracy in the maximum norm for all of the applications considered; for example, fourth-order spatial accuracy in fluid velocity, pressure, and interface location is demonstrated for surface tension-driven two phase flow in 2D and 3D. Specific application examples include: vortex shedding in nontrivial geometry, capillary wave dynamics revealing fine-scale flow features, falling rigid bodies tumbling in unsteady flow, and free surface flow over a submersed obstacle, as well as high Reynolds number soap bubble oscillation dynamics and vortex shedding induced by a type of Plateau-Rayleigh instability in water ripple free surface flow. These last two examples compare numerical results with experimental data and serve as an additional means of validation; they also reveal physical phenomena not visible in the experiments, highlight how small-scale interfacial features develop and affect macroscopic dynamics, and demonstrate the wide range of spatial scales often at play in interfacial fluid flow.

1 Introduction to Part II

In this two-part paper, a high-order accurate implicit mesh discontinuous Galerkin (dG) framework for fluid interface dynamics is developed, with the goal of enabling precise computation of fluid flow in complex geometry, e.g., to examine how small-scale interfacial features develop and affect macroscopic dynamics. The framework uses *implicitly defined meshes*—wherein a background reference quadtree/octree grid is combined with an implicit description of fluid interfaces and boundaries—and allows physically prescribed interfacial jump conditions to be imposed or captured with high-order accuracy. Several applications of the implicit mesh dG framework in two and three dimensions are presented, including examples of single phase flow in nontrivial geometry, surface tension-driven two phase flow with phase-dependent fluid density and viscosity, rigid body fluid-structure interaction, and free surface flow.

In part one of this work—see [1]—the implicit mesh dG framework is presented in detail. Here, the framework is applied to a wide array of fluid dynamics problems. As part two concentrates mainly on the governing equations of motion, time stepping methods, and experimental validation, it may be read independently of part one. However,