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A robust and efficient polyhedron subdivision and intersection algorithm for three-dimensional MMALE remapping

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ABSTRACT

The Multi-Material Arbitrary Lagrangian Eulerian (MMALE) method is an effective way to simulate the multi-material flow with severe surface deformation. Comparing with the traditional Arbitrary Lagrangian Eulerian (ALE) method, the MMALE method allows for multiple materials in a single cell which overcomes the difficulties in grid refinement process. In recent decades, many researches have been conducted for the Lagrangian, rezoning and surface reconstruction phases, but less attention has been paid to the multi-material remapping phase especially for the three-dimensional problems due to two complex geometric problems: the polyhedron subdivision and the polyhedron intersection. In this paper, we propose a "Clipping and Projecting" algorithm for polyhedron intersection whose basic idea comes from the commonly used method by Grandy (1999) [29] and Jia et al. (2013) [34]. Our new algorithm solves the geometric problem by an incremental modification of the topology based on segment–plane intersections. A comparison with Jia et al. (2013) [34] shows our new method improves the efficiency by 55% to 65% when calculating polyhedron intersections. Moreover, the instability caused by the geometric degeneracy can be thoroughly avoided because the geometry integrity is preserved in the new algorithm. We also focus on the polyhedron subdivision process and describe an algorithm which could automatically and precisely tackle the various situations including convex, non-convex and multiple subdivisions. Numerical studies indicate that by using our polyhedron subdivision and intersection algorithm, the volume conversation of the remapping phase can be exactly preserved in the MMALE simulation.

1. Introduction

Simulation of fluid with multiple-materials is a great challenge in computational mechanics and the choice of the computational grid is significant. Generally, there are two basic descriptions, the Lagrangian description and the Eulerian description. In Lagrangian description, the computational grid is embedded with material so that the material interface is always the grid boundary which can be tracked innately but the grid will be distorted when materials experience a large deformation. On the contrary, in the Eulerian description, the computational grid is fixed which overcomes the grid distortion but loses the surface information because it will advect across the cells during the simulation. In order to combine the merits of Lagrangian and Eulerian description, Hirt et al. developed an arbitrary Lagrangian–Eulerian (ALE) method [1]

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which allows the motion of grid being defined as an independent degree of freedom. In ALE frame, the best properties of Lagrangian and Eulerian descriptions are preserved and many authors have demonstrated that ALE schemes are important due to its accuracy, robustness and efficiency [2–6].

The traditional ALE scheme requires the coincidence of the material surface and cell boundary which can be easily achieved when the material surface deforms slightly. However, when it deforms severely, it is very difficult to obtain a new grid with high quality and in some extreme cases where the topology of the surface changes, it is almost impossible to perform rezoning phase successfully. To solve the problem, Peery et al. developed a multi-material ALE (MMALE) method to simulate the strong shearing flow with severe surface deformation [7]. The main improvement of MMALE method is that it allows for multiple materials in a single cell so that the material surface need not to be coincided with the cell boundary. Therefore, the MMALE method can simulate problems involving severe grid and surface deformation which are very difficult for the traditional ALE method.

The major process of the MMALE is similar to the traditional ALE method. Firstly, the Lagrangian phase is performed to update the node position and material state [8–15] but in MMALE method, a closure model [16–19] is required to determine the pressure of the mixed cells. After several Lagrangian steps, the grid will be distorted and the rezoning phase [20–25] is performed to generate a new grid with high quality. Afterwards, the remapping phase will interpolate the variables, such as the density, internal energy, velocity, from the old distorted grid to the new rezoned grid and in the MMALE method, the information for surface reconstruction, such as the volume fraction and the material centroid, should also be interpolated. Finally, we discard the distorted grid and the Lagrangian phase will be restarted again on the rezoned grid.

Generally, the remapping phase can be constructed by two approaches: the “face-based” scheme [26–28] and the “intersection-based” scheme [29–31]. These two schemes are based on the geometrical relationship between the old and the rezoned grid. The “face-based” scheme is in a flux form and the flux is determined by the relative position between the new cell’s boundary and its associated old cell’s boundary. This scheme only considers the flux across the cell face and ignores the flux from the cell corner. Therefore, the “face-based” scheme is highly efficient and can be implemented easily. However, there are two major limitations of the “face-based” scheme. 1. The rezoned grid must be in the same topology of the old grid and close to it in order to find the correct flux region. 2. In the multi-material remapping process [32], the “face-based” scheme may lead to problematic situations, such as negative mass and inaccurate material fragmentation.

On the other hand, the “intersection-based” scheme interpolates the variables by calculating the intersection portions of the old and the rezoned grid. Each intersection piece will carry the information from the old grid and then be reassembled into the rezoned grid to obtain the variables on it [28]. There are many advantages in the “intersection-based” schemes: 1. The rezoned grid need not to be close to the old grid and their topology can even be different. 2. The materials’ volume fraction and centroid for surface reconstruction can be interpolated accurately by the “intersection-based” schemes. However, the toughest difficulty in this scheme is calculating the intersection of different grids which is a very complicated geometric problem. A sampling technique was used by Horak in a two-dimensional grid intersection [31] to avoid the difficult geometric analysis but it is inaccurate and suffers from a low speed of convergence. A three-dimensional grid intersection method was proposed by Dukowicz and Padial [30], which defines the cell boundaries as a bilinear quadric surfaces and then approximate the profile of intersection by the surface–edge intersection. This algorithm runs into difficulty in some situations such as highly distorted meshes. Powell and Abel calculated the intersection of two convex polyhedrons by sequentially clipping one against the faces of the other [33], but this method is only suitable for convex polyhedrons. Grandy proposed a way to calculate the intersection between a polyhedron and a tetrahedron [29] which can precisely calculate the intersection portion through an extremely complex geometric analysis. Jia et al. simplified part of this algorithm [34]. The complexity of this algorithm will hinder the efficient assessment and it may fail in real implementation due to the geometric degeneracies [34].

Recently, some hybrid remapping methods for multi-material remapping were developed which combine the advantages of the above two basic schemes [32,35,36] but all of them are only applied in two-dimensional problem. Because the calculation of polyhedron–polyhedron intersection is significantly more complicated than polygon–polygon intersection, research on MMALE method mostly focuses on 2D problem [7,37–39]. To the best of our knowledge, there is only one public paper which involves 3D-MMALE method [34] but it does not solve the geometric problem successfully because many nonphysical material fragments occur in its 3D numerical examples. In summary, despite many accomplishments that have been made on MMALE method, efforts are still needed for a more accurate, robust, efficient and programming friendly algorithm in calculating the grid intersection especially in 3D problem.

Besides the polyhedron intersection technique, the polyhedron subdivision is another crucial issue in the multi-material remapping phase. The polyhedron subdivision will be performed after the surface reconstruction in the mixed cells to divide them into sub-polyhedrons which only contain one material. However, because of the non-convexity of the Lagrangian cells, one planar material surface may divide a mixed cell into more than two sub-polyhedrons. It is a common circumstance in 3D problems and must be precisely considered for a correct remapping phase. Moreover, if a mixed cell contains more than two materials, it will be subdivided multiple times. In summary, the non-convex subdivision and multiple subdivision cases impose difficulty in the existing subdivision algorithms, such as the clipping and capping method [40], and extra effort is required for an accurate, robust and general polyhedron subdivision algorithm.

In this paper, we firstly present an improved polyhedron subdivision algorithm which could automatically and precisely handle the convex subdivision, non-convex subdivision and multiple subdivision. Afterwards for the polyhedron intersection problem, we follow the basic idea from Grandy (1999) [29] and Jia et al. (2013) [34] but propose a different computational