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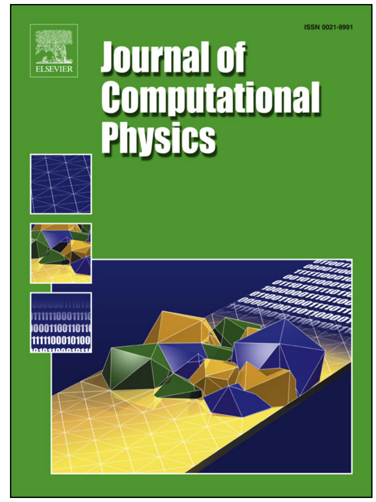

Is it possible to tune a drum?

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IS IT POSSIBLE TO TUNE A DRUM?

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Abstract. It is well known that the sound produced by string instruments has a well defined pitch. Essentially, this is due to the fact that all the resonance frequencies of the string have integer ratio with the smallest eigenfrequency. However, it is enough to use Ashbaugh-Benguria bound for the ratio of the smallest two eigenfrequencies to conclude that it is impossible to build a drum with a uniform density membrane satisfying harmonic relations on the eigenfrequencies. On the other hand, it is known since the antiquity, that a drum can produce an almost harmonic sound by using different densities, for example adding a plaster to the membrane. This idea is applied in the construction of some Indian drums like the tabla or the mridangam. In this work we propose a density and shape optimization problem of finding a composite membrane that satisfy approximate harmonic relations of some eigenfrequencies. The problem is solved by a domain decomposition technique applied to the Method of Fundamental Solutions and Hadamard shape derivatives for the optimization of inner and outer boundaries. This method allows to present new configurations of membranes, for example a two-density membrane for which the first 21 eigenfrequencies have approximate five harmonic relations or a three-density membrane for which the first 45 eigenfrequencies have eight harmonic relations, both involving some multiple eigenfrequencies.

1. Introduction

Let $\Omega \subset \mathbb{R}^2$ be a bounded domain, and consider the Dirichlet eigenvalue problem,

$$\begin{cases} -\Delta u = \rho\lambda u & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

defined in the Sobolev space $H_0^1(\Omega)$. This is a model for the vibration of a drum where the shape of the membrane is defined by the geometry of Ω. ρ is the density of the membrane and we will assume that $\rho = \rho_1\chi_{\Omega\setminus D} + \rho_2\chi_D$ and $\rho_i > 0$, i=1,2.

We will denote the eigenvalues by $0 < \lambda_1(\Omega, D, \rho) \leq \lambda_2(\Omega, D, \rho) \leq ...$ where each $\lambda_i(\Omega, D, \rho)$ is counted with its multiplicity and the corresponding orthonormal real eigenfunctions by $u_i,\ i = 1, 2, ...$ We will also use the notation $\kappa_i(\Omega, D, \rho) = \sqrt{\lambda_i(\Omega, D, \rho)}$ to denote an eigenfrequency.

In this work we will focus on the acoustic properties of a drum that can be modeled by problem (1). In particular, in this simplified model we are neglecting the body of the drum.

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Key words and phrases. harmonic drum, Dirichlet Laplacian, eigenvalues, shape optimization.

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