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Parametric lattice Boltzmann method

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ABSTRACT

The discretized equilibrium distributions of the lattice Boltzmann method are presented by using the coefficients of the Lagrange interpolating polynomials that pass through the points related to discrete velocities and using moments of the Maxwell–Boltzmann distribution. The ranges of flow velocity and temperature providing positive valued distributions vary with regulating discrete velocities as parameters. New isothermal and thermal compressible models are proposed for flows of the level of the isothermal and thermal compressible Navier–Stokes equations. Thermal compressible shock tube flows are simulated by only five on-lattice discrete velocities. Two-dimensional isothermal and thermal vortices provoked by the Kelvin–Helmholtz instability are simulated by the parametric models.

1. Introduction

One way of simulating fluid flows is to use artificial particles jumping from one node to another in a regular lattice with a limited number q of discrete velocities as in the lattice Boltzmann method [1–6]. At a given node x and time t, the existence of a particle having a given discrete velocity v_i is expressed by a probability $p_i(x,t)$ in real numbers instead of zero or one. Hence, the density of the particles having v_i is

$$f_i(x,t) = \rho(x,t)p_i(x,t) \tag{1}$$

where $\rho(x,t)$ is a total density. Particles collide with each other every time step Δt and thus velocity distributions change according to a given redistribution rule $r_i(x,t)$ or a discretized equilibrium distribution,

$$f_i^{eq}(x,t) = \rho(x,t)r_i(x,t), \tag{2}$$

within the following discretized advection formula having a single relaxation constant ω as

$$f_i(x + v_i\Delta t, t + \Delta t) = (1-\omega)f_i(x,t) + \omega f_i^{eq}(x,t). \tag{3}$$

The constitution of $f_i^{eq}(x,t)$ with corresponding discrete velocities v_i affects the accuracy, efficiency, and stability of the lattice Boltzmann method. We will present a new general form of f_i^{eq} for the purpose of simulating flows of the level of the Navier–Stokes equations

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$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho u) = 0, \\ \partial_t(\rho u) + \nabla \cdot (\rho u \otimes u) = \nabla \cdot (\mathbf{S} - \rho\theta \mathbf{I}), \\ \partial_t \left(\frac{d}{2}\rho\theta\right) + \nabla \cdot \left(\frac{d}{2}\rho\theta u\right) + \nabla \cdot \mathbf{q} = \mathbf{S} : (\nabla u) - \rho\theta \nabla \cdot u \end{cases} \tag{4}$$

with

$$\mathbf{S} = \nu\rho(\nabla u + \nabla u^{\mathsf{T}} - \frac{2}{d}\nabla \cdot u\mathbf{I}) + \eta \nabla \cdot u\mathbf{I},$$
$$\mathbf{q} = -\kappa \nabla\theta$$

where $\theta \equiv kT/m$ with k being the Boltzmann constant, T the Kelvin temperature, and m mass of a particle, u is flow velocity, d dimension of space, ν kinematic viscosity, η bulk viscosity, and κ thermal conductivity. The general form is not limited to provide models up to this level but beyond by increasing the number of discrete velocities q.

2. Parametric discretized equilibrium distribution

2.1. General form

Here, we present new discretized equilibrium distributions f_i^{eq}, namely parametric discretized equilibrium distributions. For simplicity, we present r_i that gives f_i^{eq} in one-dimensional space according to Eq. (2) as

$$r_i = \sum_{j=1}^{q} c_{ij}\mu_{j-1} \tag{5}$$

where c_{ij} is the coefficient corresponding to the term of degree $j-1$ of the Lagrange interpolating polynomial that passes through (v_k, δ_{ik}) for $k = 1, 2, \dots, q$ in which δ_{ik} is the Kronecker delta and μ_n is the nth moment of the Maxwell–Boltzmann distribution $F(v)$ defined by $\mu_n = \int v^n F(v)dv$. By defining $\hat{\mu}_n = \sum v_i^n r_i$, this rule r_i satisfies the nth moment identity $\hat{\mu}_n = \mu_n$ for $n = 0, 1, \dots, q-1$ in one-dimensional space so that we have a relation between a desired order of accuracy n^* and the number of discrete velocities q as

$$n^* = q - 1. \tag{6}$$

The detailed derivation is provided in Appendix A. Multi-dimensional models can be obtained by tensor products of one-dimensional models or be directly derived from Eq. (16) with proper choices of discrete velocities and a desired accuracy.

According to the Chapman–Enskog expansion [7,8], we obtain that a model satisfying $n^* = 3$ recovers the isothermal compressible Navier–Stokes equations, namely the first two lines of Eq. (4) with bulk viscosity $\eta = 0$ and kinematic viscosity

$$\nu = \left(\frac{1}{\omega} - \frac{1}{2}\right)\theta \Delta t \tag{7}$$

and a model satisfying $n^* = 4$ recovers the thermal compressible Navier–Stokes equations, namely Eq. (4) with the same kinematic and bulk viscosities to the isothermal model and thermal conductivity

$$\kappa = \frac{d+2}{2}\nu\rho. \tag{8}$$

2.2. Advantage of parametric models

The parametric lattice Boltzmann method (PLBM) provides a different way of deriving and a different point of view of understanding the existing models including the classic lattice Bhatnagar–Gross–Krook (LBGK) model [6]. According to the framework provided by the PLBM, one can obtain, for a given number of discrete velocities, a set of lattice Boltzmann models which are equipped with parameters. For example, considering the models of three discrete velocities, one can obtain the LBGK model by fixing the parameter $\zeta = 3$ in Eq. (9).

The new several models provided by the PLBM have advantages with respect to the existing counterpart models as the followings. One can obtain a new model with three discrete velocities, which is called the parametric model with $\zeta = 4$ in this article. This model is more stable than the LBGK model and is more accurate than the entropic model. The formula, analysis, and benchmark test are described in the following sections and especially in Eq. (9), Table 1, Figs. 1 to 7.

In addition, one can obtain a new model with four discrete velocities by the PLBM, which recovers the accuracy of the isothermal Navier–Stokes equations by the Chapman–Enskog expansion. Note that the three velocities models such as the LBGK and the entropic models do not recover the exact isothermal Navier–Stokes equations. The errors of these models are provided in Table 2. We also emphasize that the parametric four velocities models provide *on-lattice* models in contrast to the existing off-lattice ones. Details are explained in the following subsection.