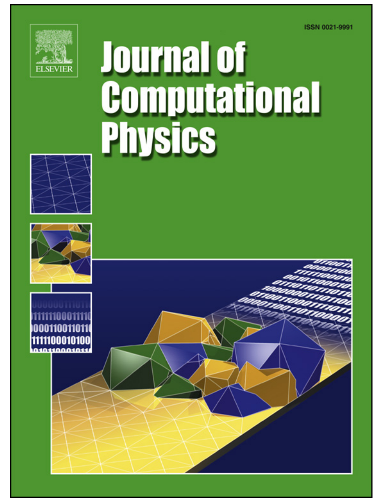

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Ken Mattsson, Martin Almquist

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A high-order accurate embedded boundary method for first order hyperbolic equations

Ken Mattsson[a,*], Martin Almquist[a]

[a] *Uppsala University, Department of Information Technology, Lägerhyddsvägen 2, 752 37 Uppsala, Sweden*

Abstract

A stable and high-order accurate embedded boundary method for first order hyperbolic equations is derived. Where the grid-boundaries and the physical boundaries do not coincide, high order interpolation is used. The boundary stencils are based on a summation-by-parts framework, and the boundary conditions are imposed by the SAT penalty method, which guarantees linear stability for one-dimensional problems. Second-, fourth-, and sixth-order finite difference schemes are considered. The resulting schemes are fully explicit. Accuracy and numerical stability of the proposed schemes are demonstrated for both linear and nonlinear hyperbolic systems in one and two spatial dimensions.

Keywords: high-order accurate, finite difference method, Cartesian grid, immersed boundaries, embedded boundaries, hyperbolic problems

1. Introduction

Hyperbolic initial-boundary value problems (IBVP) appear in many fields of science, such as acoustics, electromagnetics, and seismology. When solving hyperbolic IBVP numerically, finite difference methods have proven efficient in the sense that they approximate hyperbolic wave propagation accurately at low computational cost. Furthermore, Kreiss and Oliger [13] showed that, when the solution is smooth, high-order methods outperform low-order ones by requiring far fewer degrees of freedom for a given error tolerance. Hence, we shall strive to use a high-order finite difference method (HOFDM). One disadvantage with this approach is that with increased order of accuracy, it typically becomes more difficult to impose boundary conditions (BC) in a stable manner. Another obstacle is that HOFDM require high-quality structured grids, which may in fact be infeasible to generate in complex geometries. In such cases, one often sacrifices efficiency for flexibility by employing methods that support unstructured meshes, such as finite volume or finite element methods. An alternative approach, which we shall pursue in this paper, is to embed (or immerse) the complex geometry in a Cartesian grid, which renders the grid generation process trivial.

Embedding or immersing complex geometries in Cartesian grids is a practical technique that has gained interest during the last two decades. In the literature, different embedded boundary (EB) methods have been applied to many different PDEs. The immersed boundary (IB) method [33, 19] is the first application of Cartesian grid methods to CFD. A second order accurate IB method for turbulent flows and multi-material heat transfer problems is presented in [11, 12], where the IB method is combined with local mesh refinement. The interface between different materials can also be regarded as an immersed boundary; this technique is usually referred to as the immersed interface method [21, 20, 23, 39].

Second-order accurate EB techniques for Laplace's equation subject to Dirichlet and Neumann boundary conditions can be found in for example [6]. Several second-, fourth-, and even higher-order accurate schemes have been derived to handle embedded discontinuous coefficients in the one-dimensional case (see for example

*Corresponding author

Email addresses: ken.mattsson@it.uu.se (Ken Mattsson), martin.almquist@it.uu.se (Martin Almquist)