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A model and numerical method for compressible flows with capillary effects

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ABSTRACT

A new model for interface problems with capillary effects in compressible fluids is presented together with a specific numerical method to treat capillary flows and pressure waves propagation. This new multiphase model is in agreement with physical principles of conservation and respects the second law of thermodynamics. A new numerical method is also proposed where the global system of equations is split into several submodels. Each submodel is hyperbolic or weakly hyperbolic and can be solved with an adequate numerical method. This method is tested and validated thanks to comparisons with analytical solutions (Laplace law) and with experimental results on droplet breakup induced by a shock wave.

1. Introduction

The breakup of liquid droplets induced by high speed flows has a wide range of engineering and scientific applications and has given rise to a large number of publications. In some cases, this phenomenon causes damages as for example when droplets are impacting aircrafts in supersonic flight causing erosion of its surface (Engel [8], Joseph et al. [23], Igra and Takayama [20,21]). Studying of droplets behavior in a high speed flow may also be encountered when security issues are considered as, for example, for shock wave attenuation (Chauvin et al. [4,5]). Other applications can be found in explosive science or in combustion systems where a liquid jet is atomized (Welch and Boyle [52], Meng and Colonius [33], Devassy et al. [7]). Detailed reviews on droplet breakup can be found in Pilch and Erdman [43], Wierzba and Takayama [53], Hsiang and Faeth [18].

Concerning numerical simulations, the breakup study is usually focused on the first stages of droplet deformation when Richtmyer–Meshkov and/or Rayleigh–Taylor instabilities appear (Yang et al. [55], Quirk and Karni [44], Layes and Le Metayer [28], Meng and Colonius [33]), but not on the further stages when capillary and/or viscous effects become significant.

In the last decades, several theoretical studies have been performed to treat capillary effects in multiphase flows. The seminal work of Brackbill et al. [3] succeeded in transforming a surface force into a volume force, quite easy to treat as a source term in a multiphase flow model. The surface tension volume force is expressed thanks to a color function $\tilde{c}\,(\mathbf{x})$. This approach has been used in Chen and Doolen [6], Sussman et al. [50], Gueyffier et al. [16], Osher and Fedkiw [36,37],

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Tryggvason et al. [51], Périgaud and Saurel [39], Le Martelot et al. [29] where capillary effects are added into the momentum and the energy equations.

The aim of this work is to develop a mathematical model for fluid flows with capillary effects that is hyperbolic, verifies conservation principles and entropy inequality together with a suitable numerical method capable to treat the effect of the flow on the droplet from the short time scale when the shock wave interacts with the droplet to the long time scale when capillary effects become significant. We focus in this study on multiphase compressible fluid flows only. Viscous and heat conduction are not taken into account and will be a part of future works. Some ideas on the treatment of heat conduction in multiphase compressible flows can already be found in [40].

Section 2 presents the Brackbill et al. [3] method to treat the surface tension and a review of existing models with a conservative form of the capillary terms. In Section 3, the new model with capillary effects is presented. The model is in agreement with the conservation principles and with the second law of thermodynamics. It is shown that the model is weakly hyperbolic. It has two wave characteristics associated with the classical compression waves and two new wave characteristics associated with the capillary effects. However, for multiple contact characteristics one eigenvector is always missing. Section 4 is devoted to the building of a numerical method able to solve capillary terms in a conservative manner. The method is based on split models that are separately hyperbolic or weakly hyperbolic. These submodels are solved thanks to adequate numerical schemes. Section 5 presents the validation of the method on 2D test cases. It shows that the model and the numerical method are able to treat accurately both capillary effects and shock wave propagation. Quantitative comparisons are done with other methods based on source terms integration to show the importance of the conservative formulation. To illustrate the capabilities of the model, the aerodynamic breakup of a water column induced by a shock wave is numerically solved and is compared with experiments. In Appendix A, the model derivation is given.

2. Compressible two-phase capillary flows: state of the art

2.1. Surface tension force and color function

The main difficulty in modeling the capillary effects is about considering a surface force in numerical models that solve volume average quantities. The seminal work of Brackbill et al. [3], called CSF (Continuum Surface Force) method, succeeded to do it by using a color function, $\tilde{c}(\mathbf{x})$. Thanks to this function, the surface tension volume force is then expressed:

$$\mathbf{F}_v(\mathbf{x}) = \sigma\kappa(\mathbf{x})\frac{\nabla\tilde{c}(\mathbf{x})}{[\tilde{c}]},$$

where σ is the surface tension coefficient and $\kappa(\mathbf{x})$ the local curvature of the interface defined by:

$$\kappa(\mathbf{x}) = -\nabla\cdot\mathbf{n}(\mathbf{x}),$$

where $\mathbf{n}(\mathbf{x})$ is the normal vector to the interface between the both phases:

$$\mathbf{n}(\mathbf{x}) = \frac{\nabla\tilde{c}(\mathbf{x})}{\|\nabla\tilde{c}(\mathbf{x})\|}.$$

The color function $\tilde{c}(\mathbf{x})$ allows locations of the different fluids and the interface. $\tilde{c}(\mathbf{x})$ is defined as:

$$\tilde{c}(\mathbf{x}) = \begin{cases} c_1 & \text{in fluid 1,} \\ c_2 & \text{in fluid 2,} \\ c_1 \leqslant \tilde{c}(\mathbf{x}) \leqslant c_2 & \text{in the transition region.} \end{cases} \quad (1)$$

In the transition region $\tilde{c}(\mathbf{x})$ is given by interpolation, meaning that the interface has a non-zero thickness. $[\tilde{c}] = c_2 - c_1$ is the jump of the color function.

It is assumed that the color function obeys a transport equation [3]:

$$\frac{\partial\tilde{c}(\mathbf{x})}{\partial t} + \mathbf{u}_I\cdot\nabla\tilde{c}(\mathbf{x}) = 0,$$

where $\mathbf{u}_I$ is the interface velocity.

Numerical results using this force can be found in [6,29,36,37,50,51]. In these references, the surface tension force is treated as source terms in the momentum and the energy equations. Nevertheless, this treatment of capillary effects violates conservation principles.

2.2. Review of existing compressible models with capillary effects

Two family of methods are available to treat interface problems.