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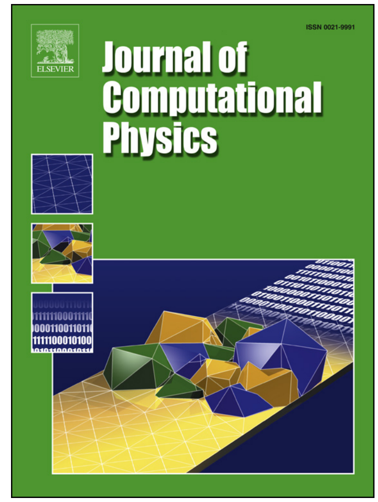

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On Consistent Boundary Closures for Compact Finite-Difference WENO Schemes

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Abstract

The accuracy of compact finite-difference schemes can be degraded by inconsistent domain or box boundary treatments. A consistent higher-order boundary closure is especially important for block-structured Cartesian AMR solvers, where the computational domain is generally decomposed into a large number of boxes containing a relatively small number of grid points. At each box boundary, a consistent higher-order boundary closure needs to be applied to avoid a reduction of the formal order-of-accuracy of the numerical scheme. This paper presents such a boundary closure for the fifth-order accurate compact finite-difference WENO scheme by Ghosh and Baeder [1]. The accuracy of the new boundary closure is validated by employing the method of manufactured solutions. A comparison of the new compact boundary closure with the original explicit boundary closure demonstrates the improved accuracy for the new compact boundary closure, while the behavior of the scheme across discontinuities appears unaffected. The linear stability analysis results indicate that a linearly stable compact WENO boundary closure is achieved.

Keywords: higher-order, compact finite-difference, weighted essentially non-oscillatory, boundary closure, adaptive mesh-refinement, shock-capturing

1. Introduction

Replacing explicit interpolation or finite-difference (FD) operators with a compact scheme (see Lele [2] for more information) is a very appealing approach for obtaining higher spectral resolution accuracy with a narrower grid stencil. In this paper, a consistent boundary closure for the fifth-order compact weighted essentially non-oscillatory (WENO) scheme by Ghosh and Baeder [1] is presented. This scheme is subsequently referred to as CWENO5. In addition to increased spectral resolution accuracy, CWENO5 provides a desirable sharp cut-off at higher frequencies, which naturally provides large dissipation for unresolved scales. In the original version of CWENO5, the compact reconstruction was combined with an explicit fifth-order reconstruction at domain boundaries. When implementing this strategy in a block-structured Cartesian AMR solver, namely the Launch Ascent and Vehicle Aerodynamics (LAVA) framework [3], it was noted that the overall accuracy of the scheme degrades with the explicit boundary treatment. Figures 1a and 1b illustrate the effect of the explicit boundary treatment on the error convergence and the imaginary part of the modified wavenumber, ω_i'. A significant improvement of the solution accuracy is noted when employing the new higher-order compact boundary closure (here referred to as "compact closure"). For solving the scalar advection equation, the computational domain with size 1×1 was decomposed into n_b boxes with $n_x = 16$ grid points. The analytical solution to the advection problem was assumed to be $u(x,t) = \sin(2\pi(x-t))$. The effect on the solution accuracy is aggravated when fewer grid points n_x are contained inside a box. The block-structured Cartesian AMR solver within LAVA is typically run with approximately $n_x = 6 - 16$ grid points. The number of grid points within a box affects the efficiency of how grid points can be clustered

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