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Simplification of the flux function for a high-order gas-kinetic evolution model

Guangzhao Zhou [a,*], Kun Xu [b], Feng Liu [c]

[a] College of Engineering, Peking University, Beijing 100871, China
[b] Department of Mathematics, Hong Kong University of Science and Technology, Kowloon, Hong Kong, China
[c] Department of Mechanical and Aerospace Engineering, University of California, Irvine, CA 92697-3975, United States

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ABSTRACT

High-order (higher-than 2nd-order) gas-kinetic schemes for solving the Navier–Stokes equations have been studied in recent years. In addition to the use of high-order reconstruction techniques, many terms are used in the Taylor expansion of the equilibrium and non-equilibrium gas distribution functions in the high-order gas kinetic flux function. Therefore, a large number of coefficients need to be determined in the calculation of the time evolution of the gas distribution function at cell interfaces. As a consequence, the high-order flux function takes much more computational time than that of a 2nd-order gas-kinetic scheme. This paper aims to simplify the evolution model by two steps. Firstly, the coefficients related to the 2nd-order spatial and temporal derivatives of a distribution function are redefined to reduce the computational cost. Secondly, based on the physical analysis, some terms can be removed without loss of accuracy. As a result, through the simplifications, the computational efficiency of the high-order scheme is increased significantly. In addition, a self-adaptive numerical viscosity is designed to minimize the necessary numerical dissipation. Several numerical examples are tested to demonstrate the accuracy and robustness of the current scheme.

1. Introduction

Most of the classical flow solvers are based on the Euler or Navier–Stokes equations. An exact or approximate Riemann solver is usually adopted for the inviscid flux. The viscous flux is treated separately from the inviscid part. However, the gas-kinetic scheme (GKS) [1,2] for the computation of compressible flows proceeds from the microscopic dynamic process. The gas distribution function is introduced to follow the gas evolution from a general initial condition in both space and time. Then all macroscopic flow variables are expressed as moments of the distribution function. Since the non-equilibrium part of the distribution function corresponds to the viscous terms, the calculations of the inviscid and viscous fluxes are performed simultaneously.

With growing demand for accurate numerical solution and continuous increase of computational power, more attention has been devoted to the development of high-order schemes in recent years. By high order here one generally refers to higher-than 2nd-order schemes. These schemes include the (weighted) essentially non-oscillatory scheme (ENO/WENO) [3,4], the discontinuous Galerkin method (DG) [5,6], the compact nonlinear scheme [7,8], and the spectral volume/difference

* Corresponding author.
E-mail addresses: zgz@pku.edu.cn (G. Zhou), makxu@ust.hk (K. Xu), fliu@uci.edu (F. Liu).

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method (SV/SD) [9,10]. For the majority of the current high-order schemes, the high-order accuracy is mostly associated with a high-order reconstruction. While the Riemann solver, which is responsible for the evolution process at the cell interface, stays the same as it is in low-order schemes. On the one hand, the Riemann solver is intrinsically one dimensional and cannot reflect the variation of variables in the second and third dimensions. Fluxes have to be evaluated at several Gaussian points on a cell interface to obtain accurate integration in the tangential direction(s) for 2-D and 3-D problems [11]. This will require disproportionally more computational cost in comparison with the 1-D case. On the other hand, the Riemann solver is a 1st-order description to the physical process and cannot reflect the variation of the status at the cell interface with time. To overcome this weakness, efforts have been taken to develop more accurate representations to the evolution process based on the generalized Riemann solver, e.g., the GRP scheme [12,13] and the ADER method [14].

Recently, the original 2nd-order gas-kinetic scheme was extended to a higher order by several authors [15–18]. In addition to the use of high-order reconstruction, the evolution process also has a high-order property, which has been shown to be important in the construction of high-order schemes [19]. With the Taylor expansions in both the perpendicular and tangential directions of a cell interface, multidimensionality is achieved and no Gaussian points are theoretically needed. Moreover, the inclusion of the time-derivatives makes it more accurate than the Riemann solver.

The previous high-order GKS schemes have shown good performance for both inviscid and viscous flows [15–18]. However, since the evolution model is associated with a large number of space- and time-dependent terms in the calculation of the flux, much more computational time is needed than that of a 2nd-order GKS. As reported in Ref. [19], with the same WENO reconstruction, the finite-volume 3rd-order GKS is 4 times slower than a finite-difference scheme with a Steger–Warming flux splitting method in the 2-D simulations. Therefore it is meaningful and necessary to reduce the computational cost of the current high-order GKS. This paper will follow the main idea in Ref. [17] for the construction of WENO-GKS. But, two simplifications are proposed to construct a more efficient scheme without loss of accuracy. The paper is organized as follows. In Section 2, the general idea of the numerical method is introduced. Section 3 is a brief review of the previous high-order gas-kinetic evolution model as the baseline model. Section 4 shows the details of the simplifications on the baseline model. The accuracy and efficiency of the simplified method are validated in Section 5 by several standard test cases, followed by results of a few other test cases of interest in Section 6. Finally, conclusions are given in Section 7.

2. Numerical procedure

We present a brief introduction to the standard procedure of gas-kinetic schemes. More details can be found in Refs. [1, 2].

The BGK equation is [20]:

$$f_t + \boldsymbol{u}\cdot\nabla f = \frac{g-f}{\tau}, \tag{1}$$

where f is the gas distribution function, g is the equilibrium distribution that f approaches, and $\boldsymbol{u} = (u, v)^T$ is the particle velocity. τ is defined as the collision time (time between collisions). It is related to the dynamic viscosity and pressure by $\tau = \mu/p$. The equilibrium function, known as the Maxwellian distribution, is

$$g = \rho\left(\frac{\lambda}{\pi}\right)^{\frac{K+2}{2}} e^{-\lambda\left[(u-U)^2+(v-V)^2+\xi^2\right]} \tag{2}$$

for 2-D flow, where ρ is the density, U, V are macroscopic velocities in x and y directions, respectively. $\lambda = m/2kT$, where m is the molecular mass, k is the Boltzmann constant and T is the temperature. K is the number of internal degrees of freedom which equals to 3 for diatomic molecules. ξ is the internal variable with $\xi^2 = \xi_1^2 + \xi_2^2 + \cdots + \xi_K^2$.

From Eqs. (1) and (2), it is clear that f is a function of $\boldsymbol{x}$, t, $\boldsymbol{u}$ and ξ. The macroscopic variables ρ, U, V and T appear as coefficients that are local constants. The conservative variables are related to the distribution function by the following equation:

$$\boldsymbol{W} = (\rho, \rho U, \rho V, \rho E)^T = \int g\boldsymbol{\psi}\, d\Xi, \tag{3}$$

where E is the total energy density.

Once the distribution function f is obtained, the flux at a cell interface normal to the x direction can be expressed as

$$\boldsymbol{F} = \int uf\boldsymbol{\psi}\, d\Xi, \tag{4}$$

where $d\Xi = dudvd\xi$, $d\xi = d\xi_1 d\xi_2 \cdots d\xi_K$, and $\boldsymbol{\psi}$ is the vector of moments:

$$\boldsymbol{\psi} = (\psi_1, \psi_2, \psi_3, \psi_4)^T = \left(1, u, v, \tfrac{u^2+v^2+\xi^2}{2}\right)^T. \tag{5}$$

In addition, we have