Journal of Computational Physics 339 (2017) 482–499

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Wavelet-based adaptation methodology combined with finite difference WENO to solve ideal magnetohydrodynamics

Seongju Do [a], Haojun Li [b], Myungjoo Kang [a,*]

[a] Department of Mathematical Sciences, Seoul National University, 151-747 Seoul, Republic of Korea
[b] School of Materials, Mechanical & Automation Engineering, Yanbian University of Science and Technology, 133000 Yanji, Jilin, People's Republic of China

ARTICLE INFO

Article history:
Received 15 July 2016
Received in revised form 7 March 2017
Accepted 13 March 2017
Available online 20 March 2017

Keywords:
Adaptive
Wavelet
WENO
Hyperbolic conservation laws
Ideal MHD equation
Euler equation

ABSTRACT

In this paper, we present an accurate and efficient wavelet-based adaptive weighted essentially non-oscillatory (WENO) scheme for hydrodynamics and ideal magnetohydrodynamics (MHD) equations arising from the hyperbolic conservation systems. The proposed method works with the finite difference weighted essentially non-oscillatory (FD-WENO) method in space and the third order total variation diminishing (TVD) Runge–Kutta (RK) method in time. The philosophy of this work is to use the lifted interpolating wavelets as not only detector for singularities but also interpolator. Especially, flexible interpolations can be performed by an inverse wavelet transformation. When the divergence cleaning method introducing auxiliary scalar field ψ is applied to the base numerical schemes for imposing divergence-free condition to the magnetic field in a MHD equation, the approximations to derivatives of ψ require the neighboring points. Moreover, the fifth order WENO interpolation requires large stencil to reconstruct high order polynomial. In such cases, an efficient interpolation method is necessary. The adaptive spatial differentiation method is considered as well as the adaptation of grid resolutions. In order to avoid the heavy computation of FD-WENO, in the smooth regions fixed stencil approximation without computing the non-linear WENO weights is used, and the characteristic decomposition method is replaced by a component-wise approach. Numerical results demonstrate that with the adaptive method we are able to resolve the solutions that agree well with the solution of the corresponding fine grid.

1. Introduction

The ideal magnetohydrodynamics (MHD) equations model the motions of a perfectly conducting plasma and provide conservation laws for the mass, momentum, total energy, as well as magnetic field. The case that the magnetic field is identically zero in the MHD equations is equivalent to the Euler equation governing the dynamics of inviscid fluids. The equations have been received as the excellent models in various applications such as weather prediction, gas dynamics, astrophysics and plasma applications. It is important to implement accurate and efficient simulations of Euler and ideal magnetodynamics (MHD) equations. Recently high order methods have received great attention from various computational

* Corresponding author.
E-mail addresses: dsj@snu.ac.kr (S. Do), hjli@yust.edu (H. Li), mkang@snu.ac.kr (M. Kang).

http://dx.doi.org/10.1016/j.jcp.2017.03.028
0021-9991/

fields. Popular classes of high order method for hyperbolic conservation laws incorporate the finite difference and finite volume WENO methods [1–3], the discontinuous Galerkin (DG) method [4–6], the spectral method [7] and so on.

Methods for adaptation of computational domains that are beneficial to simulate the physical phenomenon with vastly different spatial scales have been developed with diverse frameworks. The adaptive mesh refinement (AMR) methods with the finite difference WENO scheme [8] and the ADER-WENO (arbitrary derivative in space and time) [9–11] successfully verified the efficiency and accuracy of the AMR technique. Adaptive wavelet collocation Method (AWCM) [12–14] is one of the famous adaptation techniques. In the AWCM, the grid adaptation is based on analyzing the wavelet coefficients in the wavelet domain, and the partial differential equations are solved in physical space with the aid of finite difference methods.

It is already well known that many attractive tools of the wavelet Multi-Resolution Analysis (MRA) such as compression, denoising, edge detection and multi-scale decomposition have made themselves a promising approach in challenging researches. Actually, the AWCM has shown the potential as an adaptive numerical method by solving many different types of partial differential equations consisting of the elliptic [15], parabolic [12,14] and hyperbolic types [16,17]. Moreover, the ability of classifying the levels of grid points according to physical scales is demonstrated by simulating the Rayleigh–Taylor instability problem [18].

In this paper, we couple the AWCM framework with the fifth order FD-WENO scheme. The FD-WENO scheme is renowned for its high order accuracy, efficiency and robustness as well as its straightforward extension to multi-dimensional spaces via the dimensional splitting compared with finite volume methods. However, there is a major issue that need to be cleared. The proposed method devised on the finite difference concept, which is not the volume concept, is not fully conservative. It will be discussed in Section 5.

The outline of this paper is as follows: In Section 2 we introduce the model equations, Euler equation and ideal MHD equations. The characteristic decomposition method and the WENO interpolation algorithm are reviewed in Section 3. Section 4 is devoted to the description of the wavelet-based adaptation methodology involving the compression and reconstruction. In Section 5, we present the whole numerical process to evolve the solution and mention the issues to be addressed. In Section 6, the numerical results are implemented on several numerical examples in order to verify that the results obtained by our adaptive scheme are comparable to those of full grid scheme. Finally, conclusions and future works are given in Section 7.

2. Model equations

2.1. Euler equation

The Euler equation of gas dynamics is a non-linear system of partial differential equations described as

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho\mathbf{u}\\ E\end{pmatrix}+\nabla\cdot\begin{pmatrix}\rho\mathbf{u}\\ \rho\mathbf{u}\mathbf{u}+pI\\ (E+p)\mathbf{u}\end{pmatrix}=\begin{pmatrix}0\\ \mathbf{0}\\ 0\end{pmatrix}, \tag{1}$$

where $\rho, \mathbf{u}, p$ and E are the density, velocity, thermal pressure and total energy, respectively. Each physical quantities are connected by specifying an equation of state:

$$p=(\gamma-1)\left(E-\frac{\rho\,\|\mathbf{u}\|^2}{2}\right),$$

with a specific heat ratio γ. Here $\|\cdot\|$ indicates the Euclidean vector norm.

2.2. Ideal MHD equation

The ideal MHD equation that involves incompressible magnetic field can be described as

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho\mathbf{u}\\ E\\ \mathbf{B}\end{pmatrix}+\nabla\cdot\begin{pmatrix}\rho\mathbf{u}\\ \rho\mathbf{u}\mathbf{u}+p^*I-\mathbf{B}\mathbf{B}\\ (E+p^*)\mathbf{u}-(\mathbf{B}\cdot\mathbf{u})\mathbf{u}\\ \mathbf{u}\mathbf{B}-\mathbf{B}\mathbf{u}\end{pmatrix}=\begin{pmatrix}0\\ \mathbf{0}\\ 0\\ \mathbf{0}\end{pmatrix}, \tag{2}$$

$$\nabla\cdot\mathbf{B}=0, \tag{3}$$

where $\mathbf{B}$ is a magnetic field, the total energy and total pressure are given by

$$E=\frac{p}{\gamma-1}+\frac{1}{2}\rho\,\|\mathbf{u}\|^2+\frac{1}{2}\,\|\mathbf{B}\|^2,\quad p^*=p+\frac{1}{2}\,\|\mathbf{B}\|^2.$$

The eigen-structure of the ideal MHD equation (2) is derived in [19]. The eigenvalues of the Jacobian matrix along the unit normal direction $\mathbf{n}$ are given by