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Adaptive enriched Galerkin methods for miscible displacement problems with entropy residual stabilization

Sanghyun Lee *, Mary F. Wheeler

The Center for Subsurface Modeling, The Institute for Computational Engineering and Sciences, The University of Texas at Austin, Austin, TX 78712, United States

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ABSTRACT

We present a novel approach to the simulation of miscible displacement by employing adaptive enriched Galerkin finite element methods (EG) coupled with entropy residual stabilization for transport. In particular, numerical simulations of viscous fingering instabilities in heterogeneous porous media and Hele–Shaw cells are illustrated. EG is formulated by enriching the conforming continuous Galerkin finite element method (CG) with piecewise constant functions. The method provides locally and globally conservative fluxes, which are crucial for coupled flow and transport problems. Moreover, EG has fewer degrees of freedom in comparison with discontinuous Galerkin (DG) and an efficient flow solver has been derived which allows for higher order schemes. Dynamic adaptive mesh refinement is applied in order to reduce computational costs for large-scale three dimensional applications. In addition, entropy residual based stabilization for high order EG transport systems prevents spurious oscillations. Numerical tests are presented to show the capabilities of EG applied to flow and transport.

1. Introduction

Miscible displacement of one fluid by another in a porous medium has attracted considerable attention in subsurface modeling with emphasis on enhanced oil recovery applications [1–4]. Here flow instabilities arising when a fluid with higher mobility displaces another fluid with lower mobility is referred to as viscous fingering. The latter has been the topic of major physically driven experimental, numerical and mathematical studies for over half a century [5–12]. Recently, viscous fingering has been applied for proppant-filled hydraulic fracture propagation [13–15] for efficient transport of proppant to the tip of fractures.

The governing mathematical system that represents the displacement of the fluid mixtures consists of pressure, velocity, and concentration. Examples of numerical schemes for approximating this system include the following; continuous Galerkin [16–18], interior penalty Galerkin [19,20], finite differences [21], finite volumes [22], modified method of characteristics [23,24], mixed finite elements [25–27], and characteristic-mixed finite elements [28,29]. An effective approach that deals robustly with general partial differential equations as well as with equations whose type changes within the computational domain such as from advection dominated to diffusion dominated is discontinuous Galerkin (DG) [30–34]. DG is well suited for multi-physics applications and for problems with highly varying material properties [35,36]. Combining mixed finite elements and discontinuous Galerkin was studied in [37,38].

* Corresponding author.
E-mail addresses: shlee@ices.utexas.edu (S. Lee), mfw@ices.utexas.edu (M.F. Wheeler).

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There are three major issues with the above numerical approximations for coupling flow and transport; i) local mass balance, ii) local grid adaptivity, and iii) efficient solution algorithms for Darcy flow. It is well known that differentiating numerical approximations to obtain a flux suffers from loss of accuracy and the lack of local conservation on the existing mesh as well as yielding non-physical results for transport with this given flux. It is important to choose a numerical approximation which preserves local conservation to avoid spurious sources [39]. In addition, the complexities in implementing dynamic grid adaptations can limit the extension of schemes to realistic physical applications. Often methods which are computationally costly due to the number of degrees of freedom and lack of efficient solvers cannot be extended to higher order approximations for large-scale multi-physics problems with highly varying material properties.

In this paper, we introduce a new method for a flow and transport system, the enriched Galerkin finite element method (EG). This approach provides a locally and globally conservative flux and preserves local mass balance for transport. EG is constructed by enriching the conforming continuous Galerkin finite element method (CG) with piecewise constant functions [40–42], with the same bilinear forms as the interior penalty DG schemes. Moreover, EG has substantially fewer degrees of freedom in comparison with DG and a fast effective solver whose cost is roughly that of CG and EG can handle arbitrary orders of approximation [42,43]. An additional advantage of EG is that only those subdomains that require local conservation need to be enriched with a treatment of high order non-matching grids.

Our high order EG transport system is coupled with an entropy viscosity residual stabilization method introduced in [44] to avoid spurious oscillations near shocks. Instead of using limiters and non-oscillatory reconstructions, this method employs the local residual of an entropy equation to construct numerical diffusion, which is added as a nonlinear dissipation to the numerical discretization of the system. The amount of numerical diffusion added is proportional to the computed entropy residual. This technique is independent of mesh and order of approximation and has been shown to be efficient and stable in solving many physical problems with CG [45–49] and DG [50].

In our numerical examples, we illustrate that it is crucial to employ dynamic mesh adaptivity in order to reduce computational costs for large-scale three dimensional applications. Earlier work on adaptive local grid refinement in a variety contexts for flow and transport in porous media includes [51,17,52–56]. In this paper, we employ the entropy residual for dynamic adaptive mesh refinement to capture the moving interface between the miscible fluids. It is shown in [57,58] that the entropy residual can be used as a posteriori error indicator. Entropy residuals converge to the Dirac measures supported in the shocks as the discretization mesh size goes to zero whereas the residual of the equation converges to zero based on consistency [44]. Therefore the entropy residual is able to capture shocks more robustly than general residuals.

In summary, the novelties of the present paper are that we establish efficient and robust enriched Galerkin (EG) approximations for miscible displacement problems. We couple the high order entropy viscosity stabilization to an EG transport system and implement dynamic mesh adaptivity. In addition, we provide numerical examples to assess the performance of the scheme including viscous fingering instabilities.

The paper is organized as follows. The mathematical model is presented in Section 2. In Section 3, we formulate EG for flow and transport system with the entropy viscosity stabilization method and a global solution algorithm. Various numerical examples are reported in Section 4.

2. Mathematical model

Let $\Omega \subset \mathbb{R}^d$ be a bounded polygon (for $d = 2$) or polyhedron (for $d = 3$) with Lipschitz boundary $\partial\Omega$ and $(0, \mathbb{T}]$ is the computational time interval with $\mathbb{T} > 0$. We consider a multi-component miscible displacement system with a single phase slightly compressible flow. The advection–diffusion transport system for the miscible components i is given as

$$\frac{\partial}{\partial t}(\varphi\rho c_i) + \nabla\cdot(\rho\mathbf{u}c_i - \varphi\rho\mathbf{D}(\mathbf{u})\nabla c_i) = \tilde{q}_i,\ \text{in}\ \Omega\times(0,\mathbb{T}], \tag{1}$$

where φ is the porosity, $\mathbf{u} : \Omega \times [0, \mathbb{T}] \to \mathbb{R}^d$ is the velocity, $c_i : \Omega \times (0, T] \to \mathbb{R}$ is the advected mass fraction of the component i of the solution, and the average density ρ is defined as

$$\rho := \left(\sum_{i=1}^{N_c}\frac{c_i}{\rho_i}\right)^{-1} \tag{2}$$

with the total number of components N_c by assuming there is no volume change in mixing. For convenience, we assume only two components in our case ($N_c = 2$ and $i = 1, 2$), in particular we set $c := c_1$ and $1 - c := c_2$. This leads to solving for only one component as follows;

$$\frac{\partial}{\partial t}(\varphi\rho c) + \nabla\cdot(\rho\mathbf{u}c - \varphi\rho\mathbf{D}(\mathbf{u})\nabla c) = \tilde{q},\ \text{in}\ \Omega\times(0,\mathbb{T}], \tag{3}$$

where $\tilde{q} := \tilde{q}_1$ without loss of generality and the remaining component is obtained by the relation $c_1 + c_2 = 1$. Since the flow is assumed to be slightly compressible, the compressibility coefficient satisfies $c_F^i \ll 1$ in the relationship

$$\rho_i(p) \approx \rho_0^i(1 + c_F^i p),$$