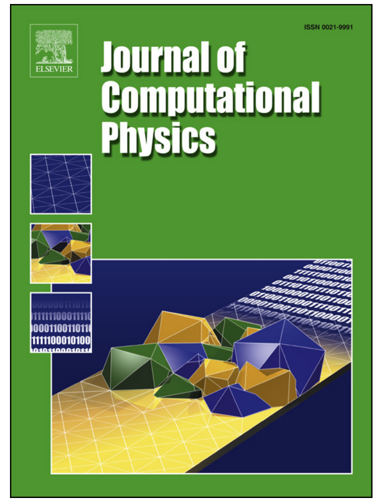
Accepted Manuscript



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PII: S0021-9991(16)30627-1
DOI: http://dx.doi.org/10.1016/j.jcp.2016.11.035
Reference: YJCPH 6985

To appear in: *Journal of Computational Physics*

Received date: 18 December 2015
Revised date: 14 October 2016
Accepted date: 25 November 2016

Please cite this article in press as: C. Batty, A cell-centred finite volume method for the Poisson problem on non-graded quadtrees with second order accurate gradients, *J. Comput. Phys.* (2016), http://dx.doi.org/10.1016/j.jcp.2016.11.035

A cell-centred finite volume method for the Poisson problem on non-graded quadtrees with second order accurate gradients

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Abstract

This paper introduces a two-dimensional cell-centred finite volume discretization of the Poisson problem on adaptive Cartesian quadtree grids which exhibits second order accuracy in both the solution and its gradients, and requires no grading condition between adjacent cells. At T-junction configurations, which occur wherever resolution differs between neighbouring cells, use of the standard centred difference gradient stencil requires that ghost values be constructed by interpolation. To properly recover second order accuracy in the resulting numerical gradients, prior work addressing block-structured grids and graded trees has shown that quadratic, rather than linear, interpolation is required; the gradients otherwise exhibit only first order convergence, which limits potential applications such as fluid flow. However, previous schemes fail or lose accuracy in the presence of the more complex T-junction geometries arising in the case of general *non-graded* quadtrees, which place no restrictions on the resolution of neighbouring cells. We therefore propose novel quadratic interpolant constructions for this case that enable second order convergence by relying on stencils *oriented diagonally* and *applied recursively* as needed. The method handles complex tree topologies and large resolution jumps between neighbouring cells, even along the domain boundary, and both Dirichlet and Neumann boundary conditions are supported. Numerical experiments confirm the overall second order accuracy of the method in the L^∞ norm.

1. Introduction

In this work, we present a new quadtree-based cell-centred finite volume technique for solving the variable coefficient Poisson problem, $\nabla \cdot \beta \nabla u = f$, over a Cartesian domain Ω in $\mathbb{R}^2$, with the domain boundary $\partial\Omega$ satisfying either Dirichlet boundary conditions, $u = g_D$, or Neumann boundary conditions, $\frac{\partial u}{\partial n} = g_N$. The solution sought is given by u, f is a source function, and the coefficient β is assumed to be positive, smoothly varying, and bounded below by some $\epsilon > 0$.

The Poisson problem has been studied extensively, and numerical techniques to solve it find wide use across computational physics, applied mathematics, engineering, and elsewhere. Cartesian grid schemes employing finite difference and finite volume discretizations on uniform grids are likewise quite well-studied (e.g., [1]). In many problems the range of scales under consideration may be quite large, which necessitates replacing simple uniform regular grids with *spatially adaptive grids*. This allows computational effort and memory usage to be focused on regions of interest, rather than being wasted on resolving insignificant areas of the domain.

A standard cell-centred uniform grid discretization of the Poisson problem provides second order accuracy of the solution variable *and its gradients*; the primary challenge in moving to the adaptive grid setting is to preserve this same behavior. Maintaining second order accurate gradients is non-trivial because changes in grid resolution introduce so-called T-junction or hanging node configurations at which the standard cell-centred finite difference/volume stencils cannot be directly applied. Nevertheless, accurate gradients can be

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