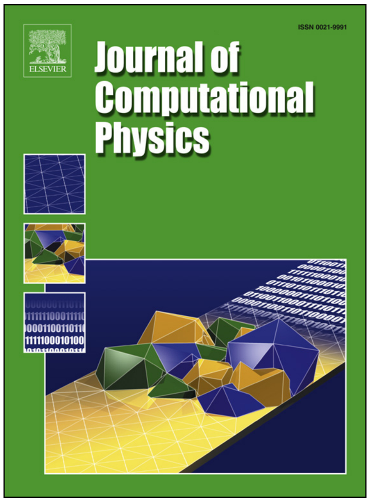

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A Study on Moving Mesh Finite Element Solution of the Porous Medium Equation

Cuong Ngo[*] Weizhang Huang[†]

Abstract

An adaptive moving mesh finite element method is studied for the numerical solution of the porous medium equation with and without variable exponents and absorption. The method is based on the moving mesh partial differential equation approach and employs its newly developed implementation. The implementation has several improvements over the traditional one, including its explicit, compact form of the mesh velocities, ease to program, and less likelihood of producing singular meshes. Three types of metric tensor that correspond to uniform and arclength-based and Hessian-based adaptive meshes are considered. The method shows a first-order convergence for uniform and arclength-based adaptive meshes and a second-order convergence for Hessian-based adaptive meshes. It is also shown that the method can be used for situations with complex free boundaries, emerging and splitting of free boundaries, and the porous medium equation with variable exponents and absorption. Two-dimensional numerical results are presented.

Key words: porous medium equation, adaptive moving mesh method, MMPDE method, finite element method, Hessian-based adaptivity, immersed boundary, free boundary

AMS subject classifications: 65M60, 65M50, 35Q35

1 Introduction

We consider the numerical solution of the initial-boundary value problem (IBVP) of the porous medium equation (PME) in two dimensions,

$$\begin{cases} u_t = \nabla \cdot (|u|^m \nabla u), & \text{in} \quad \Omega \times (t_0, T] \\ u(\boldsymbol{x}, t_0) = u_0(\boldsymbol{x}), & \text{on} \quad \Omega \\ u(\boldsymbol{x}, t) = 0, & \text{on} \quad \partial\Omega \times (t_0, T] \end{cases} \tag{1}$$

where Ω is a bounded polygonal domain, $u_0(\boldsymbol{x})$ is a given function, and $m \geq 1$ is a physical parameter. PME is a nontrivial generalization of the heat equation. It is found in many areas of the physical sciences, including gas flow in porous medium, incompressible fluid dynamics, nonlinear heat transfer, and image processing; e.g., see [55] and references therein. In the case of gas flow in porous medium, u represents the density of the gas, u^m the pressure, $u\nabla(u^m)$ the flux, $\nabla(u^m)$ the velocity, and m is the isentropic coefficient. In the case of radiation diffusion in plasmas, m stands for the power of temperature appearing in the nonlinear diffusion coefficient and can take values

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